Abstract Algebra

Principal ideal

Definition :- An ideal S of a ring R is said to be principal ideal if it is generated by a single element of S i.e. if $a \in S$ then $S = (a)$.

OR

An ideal S of R is said to be principal ideal if there exists an element $a \in S$ such that an ideal T of R containing a and also contains S.

If R is a ring with unity, then the ideal generated by 1, i.e. (1) is the ring R itself, because $r \cdot 1 = r$, $\forall\, r \in R$. Therefore, the ring R itself is called the unit ideal.

Also, the ideal generated by the zero element of R i.e. (0) is called null ideal, because (0) contains . consists of the zero element alone.

Notes :-

(i) Every ring R has at least one principal ideal, namely (0).

(ii) Every ring with unit element has at least two principal ideals namely (0) and (1)

PRINCIPAL IDEAL RING

Definition A Commutative ring R with unity having no zero divisor is called a principal ideal ring if every ideal of R is a principal ideal.

Notes :-

(i) The ring of integers is a principal ideal ring.

(ii) Every field is a principal ideal ring

DIVISIBILITY IN AN INTEGRAL DOMAIN

Definition Let a be a non-zero element of a Commutative ring R. Then a divides $b \in R$, if there exists an element $c \in R$ such that $b = ca$

we shall use the symbol $a|b$ to represents the fact that "a divides b"

In particular, if $a \neq 0$ and $0 = a \cdot 0$ then $a|0$ this implies that every non-zero element of R is a divisor of its zero element.

Units Let R be a Commutative ring with unity i.e. $1 \in R$. Then an element a of R is said to be unit in R if there exists an element $b \in R$ such that $ab = 1$

In other words, we can say that units of R are those elements of R which possess as multiplicatin

Example :- Every non-zero element in a field is a unit.

It is obvious that if a is a unit in a ring R, then a^{-1} is also a unit in R. Also the product of two units is again a unit, as if a, b are units of R, then $(ab)^{-1} = b^{-1}a^{-1} \in R$

Notes !-

(i) Do not confuse a unit with unit element of R.

(ii) There may be more than one units in a ring but the unit element is always unique.

(iii) The set of all units in R forms a group under multiplication.