is linearly independent

Theorem: Let V be an inner product space and $S = \{v_1, v_2, \dots, v_n\}$ be an orthogonal subset of V consisting of non-zero vectors. If $v \in \text{span}(S)$ then $v = \sum_{i=1}^{n} \frac{\langle v, v_i \rangle}{\|v_i\|^2} v_i$

or $y \in \text{span}(S)$ then $y = \sum_{i=1}^{n} \frac{\langle y, v_i \rangle}{\|v_i\|^2} v_i$

Proof: If $v = \sum_{i=1}^{k} \alpha_i v_i$ where $\alpha_i \in F$

$$\langle v, v_j \rangle = \left\langle \sum_{i=1}^{k} \alpha_i v_i, v_j \right\rangle \quad \text{for } 1 \le j \le k$$

$$= \sum_{i=1}^{k} \alpha_i \langle v_i, v_j \rangle$$

$$= \alpha_j \langle v_j, v_j \rangle \quad \text{for } i = j$$

$$= \alpha_j \|v_j\|^2 \Rightarrow \alpha_j = \frac{\langle v, v_j \rangle}{\|v_j\|^2}$$

or

$$\alpha_i = \frac{\langle v, v_i \rangle}{\|v_i\|^2}$$

Putting in Eq. (1)

$$v = \sum_{i=1}^{k} \frac{\langle v, v_i \rangle}{\|v_i\|^2} \cdot v_i$$

Hence the result.

Result follows

Corollary 1- If, in addition the hypothesis of theorem S is orthonormal and $v \in$ span (S) then

$$v = \sum_{i=1}^{k} \langle v, v_i \rangle v_i$$

$\because \langle v_i, v_i \rangle = 1$ for orthonormal set of vectors.

or $\|v_i\|^2 = 1$

Corollary-2 Let V be an inner product space and let S be an orthogonal subset of V consisting non-zero vectors. Then S is L.I.

i.e. $\sum_{i=1}^{k} \alpha_i \cdot v_i = 0$

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