Paper 7, TDC Part-3
Chapter– 3, Number Systems and Codes
Electronics
Lecture - 2

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Number Systems and Codes

- **<u>Weighting Structure of Binary Numbers : -</u>**

Binary number is a weighted number. The weight depends on the number of bits present in binary number and increase from right to left by a power of 2 for each bit.

The LSB of binary whole number has a weight of $2^0 = 1$ while the weight of MSB depends on the number of bits of the binary number.

Fractional numbers are written by placing bits to the right of the binary point (radix). The left most bit is the MSB and has a weight of $2^{-1} = 0.5$, the weight decrease from left to right by a negative power of 2 for each bit.

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The weight structure of binary number is

2^{n-1} …… 2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2} 2^{-3} ……… 2^{-m} 2

Where n and m are positive integers.

- **<u>Conversion of number systems</u>**

In a digital system, any user provide input (data) and receives the output data in decimal number system while the digital system understand the binary data, so it is required to convert binary number to decimal and vice versa.

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- **Binary-to-Decimal Conversion**

Any binary number can be converted to its decimal equivalent using the weights assigned to each bit position. The equivalent decimal value of any binary number is obtained by adding the weights of all bits that are 1 and discarding the weights of the bits those are zero.

Refer to solved examples in next slide.

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Example :- (1) Convert the following binary numbers to it's equivalent decimal numbers.

(i) 1110101 (ii) 0.01101 (iii) 1101.1001

Solution :- (i) Binary Number : 1 1 1 0 1 0 1
Weight : 2^6 2^5 2^4 2^3 2^2 2^1 2^0

So decimal equivalent of Binary Number ~~(1110101) =~~

$$\overset{2^6\,2^5\,2^4\,2^3\,2^2\,2^1\,2^0}{(1110101)_2} = 1\times 2^6 + 1\times 2^5 + 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

$$= 1\times 64 + 1\times 32 + 1\times 16 + 0\times 8 + 1\times 4 + 0\times 2 + 1\times 1$$

$$= 64 + 32 + 16 + 8 + 4 + 1$$
$$= (125)_{10}$$

So $(1110101)_2 = (125)_{10}$

Note :- To differentiate between numbers represented in different number systems, either the corresponding

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number system may be specified along with the number or a small subscript at the end of the number may be added signifying the number system.

Like $(100)_2$ represents a binary number and is not one hundred. Similarly $(100)_8$ & $(100)_{16}$ represent a octal number and a hexadecimal number respectively.

Solution (ii:) $(0.01101)_2 = (?)_{10}$

Fractional Binary Number :- $0.\ 0\ 1\ 1\ 0\ 1$

Weight : $2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ 2^{-5}$

So,

$$(0.\overset{2^{-1}}{0}\overset{2^{-2}}{1}\overset{2^{-3}}{1}\overset{2^{-4}}{0}\overset{2^{-5}}{1})_2 = 0\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3} + 0\times 2^{-4} + 1\times 2^{-5}$$

$$= 0\times 0.5 + 1\times 0.25 + 1\times 0.125 + 0\times 0.0625 + 1\times 0.03125$$

$$= 0 + 0.25 + 0.125 + 0 + 0.03125$$

$$= (0.40625)_{10}$$

So $(0.01101)_2 = (0.40625)_{10}$

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iii) $(1101.1001)_2 = (?)_{10}$

Binary Number: 1 1 0 1 . 1 0 0 1

Weight : 2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2} 2^{-3} 2^{-4}

So

$$(\overset{2^3}{1}\ \overset{2^2}{1}\ \overset{2^1}{0}\ \overset{2^0}{1}.\overset{2^{-1}}{1}\ \overset{2^{-2}}{0}\ \overset{2^{-3}}{0}\ \overset{2^{-4}}{1})_2 = 1\times2^3 + 1\times2^2 + 0\times2^1$$

$$+ 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 0\times2^{-3} + 1\times2^{-4}$$

$$= 1\times8 + 1\times4 + 0\times2 + 1\times1 + 1\times0.5 + 0\times0.25 + 0\times0.125 + 1\times0.0625$$

$$= 8 + 4 + 0 + 1 + 0.5 + 0 + 0 + 0.0625$$

$$= (13.5625)_{10}$$

So $(1101.1001)_2 = (13.5625)_{10}$

Example (2) $(11001.1101)_2 = (?)_{10}$

Solution $(11001.1101)_2 = 1\times2^4 + 1\times2^3 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 1\times2^{-4}$

$$= 16 + 8 + 1 + 0.5 + 0.25 + 0.0625$$

$$= (25.8125)_{10} \text{ ans}$$

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Decimal-to-Binary Conversion :→

Method 1 ⇒ Continuous (Repeated) division by 2 Method.

For integers, the conversion is obtained by continuous division by 2 and keeping the track of remainders. The remainders generated by each division form the binary number. The 1st remainder to be produced is the LSB and the last remainder to be produced is the MSB in the binary number. It can be illustrated by the following example.

Example 3) Convert decimal number 39 to it's equivalent binary number.

Solution

	Quotient	Remainder	
2	39	1	← LSB
2	19	1	
2	9	1	
2	4	0	
2	2	0	
2	1	1	← MSB
	0		

Thus = $(39)_{10} = (1\,0\,0\,1\,1\,1)_2$

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Example (4) $(63)_{10} = (?)_2$

Solution:-

	Quotient	Remainder	
2	63	1	LSB
2	31	1	
2	15	1	
2	7	1	
2	3	1	
2	1	1	MSB
	0		

$(63)_2 = (1\ 1\ 1\ 1\ 1\ 1)$

Method 2:- Sum-of-Weights Method

In this method the conversion is obtained by determing the ~~the~~ set of binary weights whose sum is equal to the decimal number.

Example (3) $(39)_{10} = (?)_2$

So we start with the weight that is just less than the given decimal number.

$39 = 32 + 4 + 2 + 1$

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$= 2^5 + 2^2 + 2^1 + 2^0$

$= 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$

So $(39)_{10} = (100111)_2$

Example (6) $(96)_{10} = (\ ?\ )_2$

Solution $(96)_{10} = 64 + 32$

$= 1\times2^6 + 1\times2^5 + 0\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0$

$(96)_{10} = (1100000)_2$

Converting Fractional Decimal to Binary

The fractional Decimal can be converted to it's equivalent binary number by continuous multiplication by 2 and keeping track of the intergers generated.

Example

Example 7) Convert $(0.65625)_{10}$ to its equivalent binary.

Solution

$0.65625 \times 2 = 1.31250 \longrightarrow 1 \longrightarrow$ MSB

$0.31250 \times 2 = 0.62500 \longrightarrow 0$

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$0.625 \times 2 = 1.250 \longrightarrow 1$

$0.250 \times 2 = 0.500 \longrightarrow 0$

$0.5 \times 2 = 1.0 \longrightarrow 1 \rightarrow$ LSB

So,

$(0.65625)_{10} = (1\,0\,1\,0\,1)$

Example :- (8) Convert $(0.7341)_{10}$ to it's equivalent binary number upto 4 binary bit after binary point.

Solution

$0.7341 \times 2 = 1.4682 \longrightarrow 1 \rightarrow$ MSB

$0.4682 \times 2 = 0.9364 \longrightarrow 0$

$0.9364 \times 2 = 1.8728 \longrightarrow 1$

$0.8728 \times 2 = 1.7456 \longrightarrow 1 \rightarrow$ LSB

So, upto 4 binary bit after binary point.

$(0.7341)_{10} = (0.1011\ldots)$

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Example 9: → $(137.65)_{10} = (?)_2$ upto 4 binary bit after binary point.

Solution: Here we first find binary equivalent of integer part, i.e. 137

2	137	1 → LSB
2	66	0
2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1 → MSB
	0	

$(137)_{10} = (10001001)_2$

Now for integer part we have.

$0.65 \times 2 = 1.30 \longrightarrow 1 \rightarrow$ MSB

$0.30 \times 2 = 0.60 \longrightarrow 0$

$0.6 \times 2 = 1.2 \longrightarrow 1$

$0.2 \times 2 = 0.4 \longrightarrow 0 \rightarrow$ LSB

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So the binary equivalent of

$$(137.65)_{10} = (10000101\ .\ 1010.....)_2$$

Thank You