GROUP THEORY

NORMALIZER OF AN ELEMENT :-

The normalizer of an element $a \in G$ is the set of all those elements of G which commutes with a and is denoted by $N(a)$ i.e.

$$N(a) = \{x \in G : ax = xa\}$$

Notes :-

(i) $N(a)$ is a subgroup of G.

(ii) In general, $N(a)$ is not subgroup of G.

(iii) $N(e) = G$ [$\because ex = xe \ \forall x \in G$]

(iv) $N(a) = G$ iff G is abelian.

Centralizer :- Let A be a non-empty subset of a group G. The Centralizer $C(A)$ of A. in G is defined as

$$C(A) = \{x \in G : ax = xa \ \forall \ a \in A\}$$

Notes :- (i) $C(A)$ is a subgroup of G.

(ii) $CA \subset N(A)$

(iii) The abelian part of a group is defined as the Centre of the group.

Commutator :- Let G be a group and $x, y \in G$. The element $x^{-1}y^{-1}xy$ is called the Commutator of x and y taken in this order.

Notes :-

(i) The inverse of a Commutator is a Commutator

(ii) The product of two Commutator is not necessary a Commutator.

Conjugate class :- Since an equivalence relation defined on a set decomposes the set into mutually disjoint equivalence classes, hence the relation of Conjugacy, defined on a group G decomposes G into mutually disjoint equivalence classes known as classes of conjugate element.

Let C_a denote the equivalence class determined by an element $a \in G$, then

$$C_a = \{x \in G : x = a\} = \{x \in G : x = y^{-1}ay \text{ for some } y \in G\}$$

$$= \{y^{-1}ay : y \in G\}$$

Here, C_a is defined as Conjugate class of a in G. Also, C_a is the class element Conjugate to a.