Nonlinear Equations:-

There is a first order differential nonlinear (i.e not first degree but higher degree) equation of the form

$$F(x, y, p) = 0 \quad \text{———} \quad (1)$$

where $p \equiv y' = \frac{dy}{dx}$ and F is a function of three variables.

Theorem:- There exists a unique solution $y = y(x)$, $x_0 - h \leq x \leq x_0 + h$ (where h is small) of Eq^n (1) that satisfies the condition $y(x_0) = y_0$ for which $y'(x_0) = y_1$, where y_1 is one of the real roots of the equation $F(x_0, y_0, y_1) = 0$ if in a closed neighbourhood of a point (x_0, y_0, y_1). The function F possesses the properties:-

(i) $F(x, y, p)$ is a continuous in all arguments

(ii) the derivative $F_p'(x, y, p)$ exists and is nonzero

(iii) there exists the derivative $|F_y'(x, y, p)| < M$.

Any solution of Eq^n (1) of the form $y = \varphi(x, c)$, where c is an arbitrary constant is called a general solution of the equation.

Any solution that may be obtained from the general solution of eph(1) by assigning particular value to the constant C is called a particular solution of that equation.

A solution of Eph(1) that cannot be obtained by assigning specific value to the arbitrary constant in the general solution is called a singular solution of that equation.

In order to solve of equation (1) one should first investigate

1-) whether it is solvable for variable for Example.

$$F(x,y,p) = y - \log(xy') + x^2y' = 0 \quad \text{then}$$

$$y = \log(xy') - x^2y'$$

2-) or it is unsolvable for every variable for examp.

$$F(x,y,p) = xy - \log(xy') + \sin(yy') = 0$$

Then in the first case 1-) one should investigate

1.i) whether all obtained functions are real

1.ii) or some obtained functions ar complex.

Finally, in the case 1. i) the following cases are possible.

a) Eph(1) is solvable for $y'(x)$

b) Eph(1) is solvable for $y(x)$

c) Eph(1) is solvable for x