# TOPIC: REFLECTION AND REFRACTION of EMW

UG-II

Under this topic, we shall observe the electro-magnetic wave going from one media to second media. This wave is called incident wave. When an incident wave strikes the interface of two different media, some part of it is reflected & other is refracted depending upon the nature of the interface. The reflected and refracted wave have both kinetic and dynamic properties.

Let us consider a plane interface at $z=0$ separating two homogeneous media characterised by permittivities $\epsilon_1$ & $\epsilon_2$ and permeabilities $\mu_1$ and $\mu_2$ respectively as shown in fig (1).

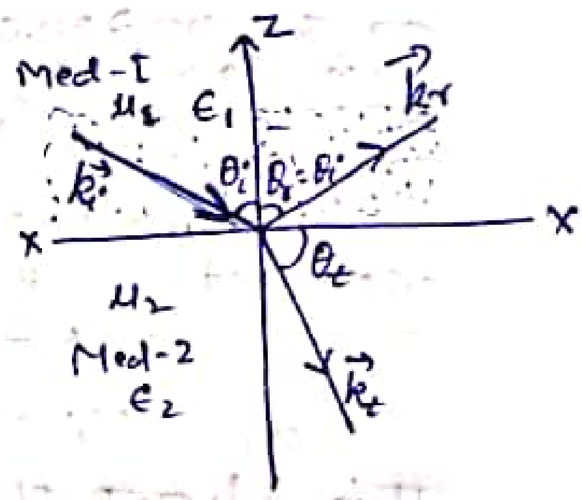


Fig (1)

Let a plane wave with wave vector $\vec{k}_i$ and $\omega_i$ be incident from medium I at point O on the interface. Let the reflected and refracted waves have wave vectors $\vec{k}_r$ and $\vec{k}_t$ and frequencies $\omega_r$ and $\omega_t$ respectively.

The time dependent electric vector of an electromagnetic wave is given by

$$E = E_0 \, e^{-i(\omega t - \vec{k}\cdot\vec{r})} \longrightarrow (1)$$

Applying the given boundary Condition, we have

$$(E_i)_{tangential} + (E_r)_{tangential} = (E_t)_{tangential}$$

or, $$(E_{0i})_x e^{-i(\omega_i t - \vec{k}_i \cdot \vec{r})} + (E_{0r})_x e^{-i(\omega_r t - \vec{k}_r \cdot \vec{r})} = (E_{0t})_x e^{-i(\omega_t t - \vec{k}_t \cdot \vec{r})} \longrightarrow (2)$$

Eq$^n$ (2) is valid for all values of $x$, $y$ and $t$ as $z$ Component is normal to the boundary. This can be satisfied if the time and space varying components of the phase are equal.

Equating the time varying components, we get,

$$\omega_i t = \omega_r t = \omega_t t$$

or, $$\omega_i = \omega_r = \omega_t = \omega \text{ (say)} \longrightarrow (3)$$

This shows that all the three waves have same frequency

Equating the space varying components in equation (2), we have

$$(\vec{k}_i \cdot \vec{r})_{z=0} = (\vec{k}_r \cdot \vec{r})_{z=0} = (\vec{k}_t \cdot \vec{r})_{z=0} \longrightarrow (4)$$

i.e, $k_{ix}x + k_{iy}y = k_{rx}x + k_{ry}y = k_{tx}x + k_{ty}y$

or $$k_{ix} = k_{rx} = k_{tx} \quad \text{------} (5)$$

and $$k_{iy} = k_{ry} = k_{ty} \quad \text{------} (6)$$

Since the incident beam is in $xz$ plane, $k_{iy} = 0$ i.e $k_{ry}$ and $k_{ty}$ are also zero. This shows that both $\vec{k}_r$ and $\vec{k}_t$ lie in xz-plane. Let $\vec{n}$ be the unit vector normal to the interface and directed from medium I into medium II i.e, normal $\vec{n}$ is along z axis. Hence all the three wave vectors and normal to the interface $\vec{n}$ all lie in the same plane. In other words the incident, reflected, refracted waves and the normal to the interface all lie in the same plane.

Now all vectors $\vec{k}_i$, $\vec{k}_r$ and $\vec{k}_t$ lie in x-z plane so that

$$\left.\begin{aligned}\vec{r} &= \vec{i}x + \vec{k}z\\ \vec{k}_i &= \vec{i}k_i\sin\theta_i + \vec{k}k_i\cos\theta_i\\ \vec{k}_r &= \vec{i}k_r\sin\theta_r - \vec{k}k_r\cos\theta_r\\ \vec{k}_t &= \vec{i}k_t\cos\theta_t + \vec{k}k_t\sin\theta_t\end{aligned}\right\} \quad (7)$$

$$\therefore \left.\begin{aligned}\vec{k}_i\cdot\vec{r} &= k_i(x\sin\theta_i + z\cos\theta_i)\\ \vec{k}_r\cdot\vec{r} &= k_r(x\sin\theta_r - z\cos\theta_r)\\ \vec{k}_r\cdot\vec{r} &= k_t(x\sin\theta_t + z\sin\theta_t)\end{aligned}\right\} \quad (8)$$

from eq$^n$(4) and (8), we have

$$k_i x\sin\theta_i = k_r x\sin\theta_r$$

$$\therefore \frac{\sin\theta_i}{\sin\theta_r} = \frac{k_r}{k_i} = \frac{\omega_r}{v_r}\times\frac{v_i}{\omega_i} = 1$$ as $\omega_i = \omega_r$ & $v_i = v_r$ in the same medium.

$$\therefore \theta_i = \theta_i \longrightarrow (9)$$

And, $k_i x\sin\theta_i = k_t x\sin\theta_t$

$$\therefore \frac{\sin\theta_i}{\sin\theta_t} = \frac{k_t}{k_i} = \frac{\omega_t}{v_t}\cdot\frac{v_i}{\omega_i} = \frac{v_i}{v_t} = \frac{n_2}{n_1} \longrightarrow (10)$$

Eq$^n$ (8) and (9) give the law of reflection and law of refraction of emw.