Langat Singh College, Muzaffarpur
NAAC Grade 'A'
Under B. R. A. Bihar University, Muzaffarpur

# Motion of charged particles–L - 07

**Dr. Tarun Kumar Dey**
**Professor in Physics**
**HOD, Electronics**
**Online Platform: https://meet.findmementor.com**

# Non Uniform E (Finite Larmor Radius)

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E}(\mathbf{r}) + \mathbf{v}\wedge\mathbf{B}\right) \tag{91}$$

Seek the usual solution $v = v_D + v_g$.

Then average out over a gyro orbit

$$\langle m\frac{dv_D}{dt}\rangle = 0 = \langle q\left(\mathbf{E}(\mathbf{r}) + \mathbf{v}\wedge\mathbf{B}\right)\rangle \tag{92}$$

$$= q\left[\langle\mathbf{E}(\mathbf{r})\rangle + \mathbf{v}_D\wedge\mathbf{B}\right] \tag{93}$$

Hence drift is obviously

$$\mathbf{v}_D = \frac{\langle \mathbf{E}(\mathbf{r}) \rangle \wedge \mathbf{B}}{B^2} \qquad (94)$$

So we just need to find the average E field experienced.

Expand E as a Taylor series about the G.C.

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 + (\mathbf{r}.\nabla)\,\mathbf{E} + \left(\frac{x^2\partial^2}{2!\partial x^2} + \frac{y^2}{2!}\frac{\partial^2}{\partial y^2}\right)\mathbf{E} + \text{cross terms } + . \qquad (95)$$

$$\text{(E.g. cross terms are } xy\frac{\partial^2}{\partial x\,\partial y}\mathbf{E}) \qquad (96)$$

linear term $\langle rL \rangle = 0$. $S_o$

$$\langle \mathbf{E}(\mathbf{r}) \rangle \simeq \mathbf{E} + \frac{r_L^2}{4}\nabla^2 E \tag{97}$$

Hence E ∧ B with 1st finite-Larmor-radius correction is

$$\mathbf{v}_{E\wedge B} = \left(1 + \frac{r_L^2}{r}\nabla^2\right)\frac{\mathbf{E}\wedge\mathbf{B}}{B^2}. \tag{98}$$

[Note: Grad B drift is a finite Larmor effect already.]

Second and Third Adiabatic Invariants

There are additional approximately conserved quantities like μ in some geometries.

# Summary of Drifts

$$\mathbf{v}_E = \frac{\mathbf{E} \wedge \mathbf{B}}{B^2} \qquad \text{Electric Field} \qquad (99)$$

$$\mathbf{v}_F = \frac{1}{q}\frac{\mathbf{F} \wedge \mathbf{B}}{B^2} \qquad \text{General Force} \qquad (100)$$

$$\mathbf{v}_E = \left(1 + \frac{r_L^2}{4}\nabla^2\right)\frac{\mathbf{E} \wedge \mathbf{B}}{B^2} \qquad \text{Nonuniform E} \qquad (101)$$

$$\mathbf{v}_{\nabla B} = \frac{mv_\perp^2}{2q}\frac{\mathbf{B} \wedge \nabla B}{B^3} \qquad \text{GradB} \qquad (102)$$

$$\mathbf{v}_R = \frac{mv_\parallel^2}{q}\frac{\mathbf{R}_c \wedge \mathbf{B}}{R_c^2 B^2} \qquad \text{Curvature} \tag{103}$$

$$\mathbf{v}_R + \mathbf{v}_{\nabla B} = \frac{1}{q}\left(mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right)\frac{\mathbf{R}_c \wedge \mathbf{B}}{R_c^2 B^2} \qquad \text{Vacuum Fields.} \tag{104}$$

$$\mathbf{v}_p = \frac{q}{|q|}\frac{\dot{E}_\perp}{|\Omega| B} \qquad \text{Polarization} \tag{105}$$

# Mirror Motion

$$\mu \equiv \frac{mv_\perp^2}{2B} \qquad \text{is constant} \tag{106}$$

Force is $F = -\mu \nabla B$.