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Motion of Charged Particles–L 06

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Time Varying E-field (E, B uniform)

Recall the E ∧ B drift:

$$\mathbf{v}_{E\wedge B} = \frac{\mathbf{E}\wedge\mathbf{B}}{B^2} \tag{74}$$

when E varies so does $v_{E\wedge B}$. Thus the guiding centre experiences acceleration

$$\dot{\mathbf{v}}_{E\wedge B} = \frac{d}{dt}\left(\frac{\mathbf{E}\wedge\mathbf{B}}{B^2}\right) \tag{75}$$

In the frame of the guiding centre which is accelerating, a force is felt.

$$\mathbf{F}_a = -m\frac{d}{dt}\left(\frac{\mathbf{E}\wedge\mathbf{B}}{B^2}\right) \qquad \text{(Pushed back into seat! } - \text{ve.)} \tag{76}$$

This force produces another drift

$$\mathbf{v}_D = \frac{1}{q}\frac{\mathbf{F}_a \wedge \mathbf{B}}{B^2} = \frac{m}{qB^2}\frac{d}{dt}\left(\frac{\mathbf{E}\wedge\mathbf{B}}{B^2}\right)\wedge\mathbf{B} \qquad (77)$$

$$= -\frac{m}{qB}\frac{d}{dt}\left((\mathbf{E}.\mathbf{B})\mathbf{B} - B^2\mathbf{E}\right) \qquad (78)$$

$$= \frac{m}{qB^2}\dot{\mathbf{E}}_\perp \qquad (79)$$

This is called the 'polarization drift'.

$$\mathbf{v}_D = \mathbf{v}_{E\wedge B} + \mathbf{v}_p = \frac{\mathbf{E}\wedge\mathbf{B}}{B^2} + \frac{m}{qB^2}\dot{\mathbf{E}}_\perp \qquad (80)$$

$$= \frac{E\wedge B}{B^2} + \frac{1}{\Omega B}\dot{\mathbf{E}}_\perp \qquad (81)$$

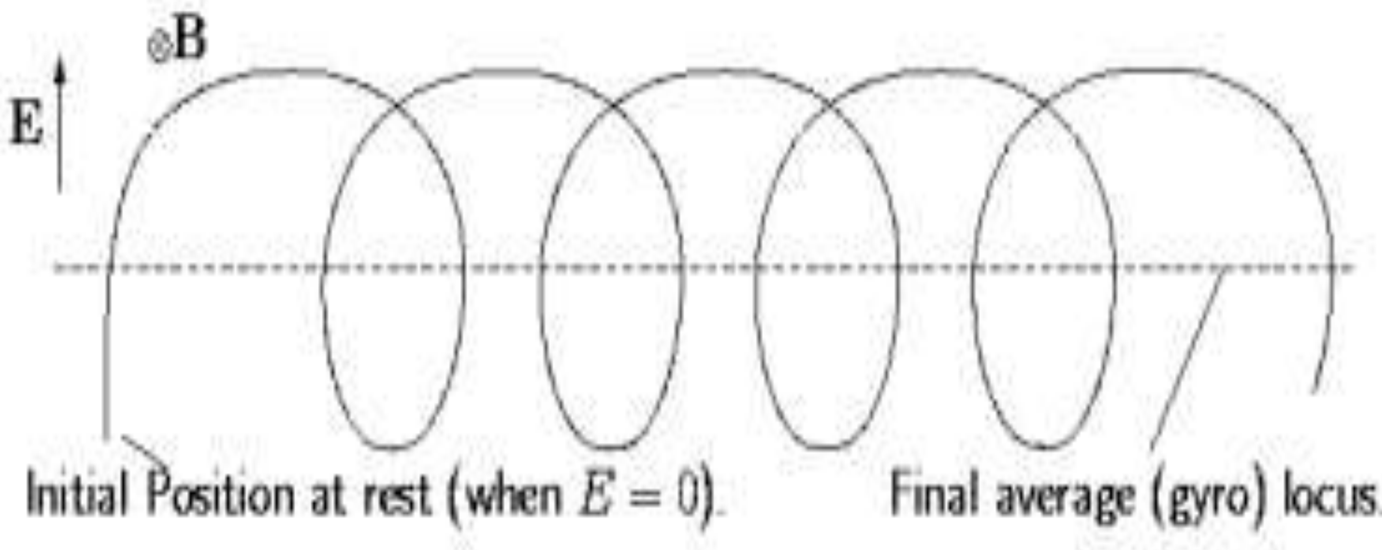

Figure 12. Suddenly turning on an electric field causes a shift of the gyrocenter in the direction of force. This is the polarization drift.

Startup effect: When we ‘switch on’ an electric field the average position (gyro center) of an initially stationary particle shifts over by 1/2 the orbit size. The polarization drift is this polarization effect on the medium. Total shift due to v_p is

$$\Delta \mathbf{r} \int \mathbf{v}_p dt = \frac{m}{qB^2} \int \hat{\mathbf{E}}_\perp dt = \frac{m}{qB^2} [\Delta \mathbf{E}_\perp] \qquad (82)$$

Direct Derivation of $\frac{dE}{dt}$ effect: ‘Polarization Drift’

Consider an oscillatory field $E = E\ _{e}{-}^{i\omega t}$ $(\perp r_0 B)$

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}\right) \quad (83)$$

$$= q\left(\mathbf{E}e^{-i\omega t} + \mathbf{v} \wedge \mathbf{B}\right) \quad (84)$$

Try for a solution in the form

$v = v_D e{-}^{i\omega t} + v_L$ (85)

where, as usual, v_L satisfies $m_L = qv_L \wedge B$

Then

$$(1)\quad m(-i\omega v_D = q\,(E + v_D \wedge B) \quad x\,e^{-iwt} \quad (86)$$

Solve for v_D : Take $\wedge$B this equation:

$$(2) \quad -mi\omega(\mathbf{v}_D \wedge \mathbf{B}) = q\left(\mathbf{E}\wedge\mathbf{B} + \left(\mathbf{B}^2.\mathbf{v}|D\right)\mathbf{B} - B^2\mathbf{v}_D\right) \tag{87}$$

Add miω × (1) to q × (2) to eliminate $v_D \wedge$ B.

$$m^2\omega^2\mathbf{v}_D + q^2(\mathbf{E}\wedge\mathbf{B} - B^2\mathbf{v}_D) = mi\omega q\mathbf{E} \tag{88}$$

or

$$\mathbf{v}_D\left[1 - \frac{m^2\omega^2}{q^2B^2}\right] = -\frac{mi\omega}{qB^2}\mathbf{E} + \frac{\mathbf{E}\wedge\mathbf{B}}{B^2} \tag{89}$$

$$\text{i.e.} \quad \mathbf{v}_D\left[1 - \frac{\omega^2}{\Omega^2}\right] = -\frac{i\omega q}{\Omega B|q|}\mathbf{E} + \frac{\mathbf{E}\wedge\mathbf{B}}{B^2} \tag{90}$$

Since –iω ↔ $\frac{\partial}{\partial t}$ this is the same formula as we had before: the sum of polarization and ∂t E ∧ B drifts ↔ except for the $[1 - \omega^2\Omega^2]$ term

This term comes from the change in v_D with time (accel).

Thus our earlier expression was only approximate. A good approx if $\omega << \Omega$.