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NAAC Grade 'A'
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Motion of charged particles– L - 04

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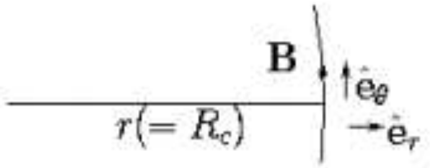
Vacuum Fields

Relation between ∇B & R_c drifts

The curvature and ∇B are related because of Maxwell's equations, their relation depends on the current density J. A particular case of interest is J = 0: vacuum fields

Figure 7. Local polar coordinates in a vacuum field



$\nabla \wedge B = 0$ (static case) (46)

Consider the z-component

$$0 = (\nabla \wedge \mathrm{B})_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) \quad (B_r = 0 \text{ by choice}). \qquad (47)$$

$$= \frac{\partial B_\theta}{\partial r} + \frac{B_\theta}{r} \qquad (48)$$

or, in other words,

$$(\nabla B)_r = -\frac{B}{R_c} \qquad (49)$$

[Note also $0 = (\nabla \wedge B)_\theta = \partial B_\theta/\partial_z$: $(\nabla B)_z = 0$]

and hence $(\nabla B)_{perp} = -B\, R_c/R_c^2$

Thus the grad B drift can be written:

$$\mathbf{v}_{\nabla B} = \frac{mv_\perp^2}{2q}\frac{\mathbf{B}\wedge\nabla B}{B^3} = \frac{mV_\perp^2}{2q}\frac{\mathbf{R}_c\wedge\mathbf{B}}{R_c^2B^2} \qquad (50)$$

and the total drift across a vacuum field becomes

$$\mathbf{v}_R + \mathbf{v}_{\nabla B} = \frac{1}{q}\left(mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right)\frac{\mathbf{R}_c\wedge\mathbf{B}}{R_c^2B^2}\,. \qquad (51)$$

Notice the following:

1. R_c & ∇B drifts are in the same direction.

2.They are in opposite directions for opposite charges

3.They are proportional to particle energies

4. Curvature $\leftrightarrow$ Parallel Energy ($\times$ 2) ∇B $\leftrightarrow$ Perpendicular Energy

5. As a result one can very quickly calculate the average drift over a thermal distribution of particles because

$$\langle \frac{1}{2} m v_{\parallel}^2 \rangle = \frac{T}{2} \tag{52}$$

$$\langle \frac{1}{2} m v_{\perp}^2 \rangle = T \quad \text{2 degrees of freedom} \tag{53}$$

Therefore

$$\langle \mathbf{v}_R + \mathbf{v}_{\nabla B} \rangle = \frac{2T}{q} \frac{\mathbf{R}_c \wedge \mathbf{B}}{R_c^2 B^2} \left(= \frac{2T}{q} \frac{\mathbf{B} \wedge \left(\hat{\mathbf{b}} . \nabla \right) \hat{\mathbf{b}}}{B^2} \right) \quad (54)$$

Mirror Trapping

$F_{||}$ may be enough to reflect particles back. But may not!

Let's calculate whether it will: Suppose reflection occurs.

At reflection point $v_{||r} = 0$. Energy conservation

$$\frac{1}{2} m (v_{\perp 0}^2 + v_{\parallel 0}^2) = \frac{1}{2} m v_{\perp r}^2 \quad (55)$$

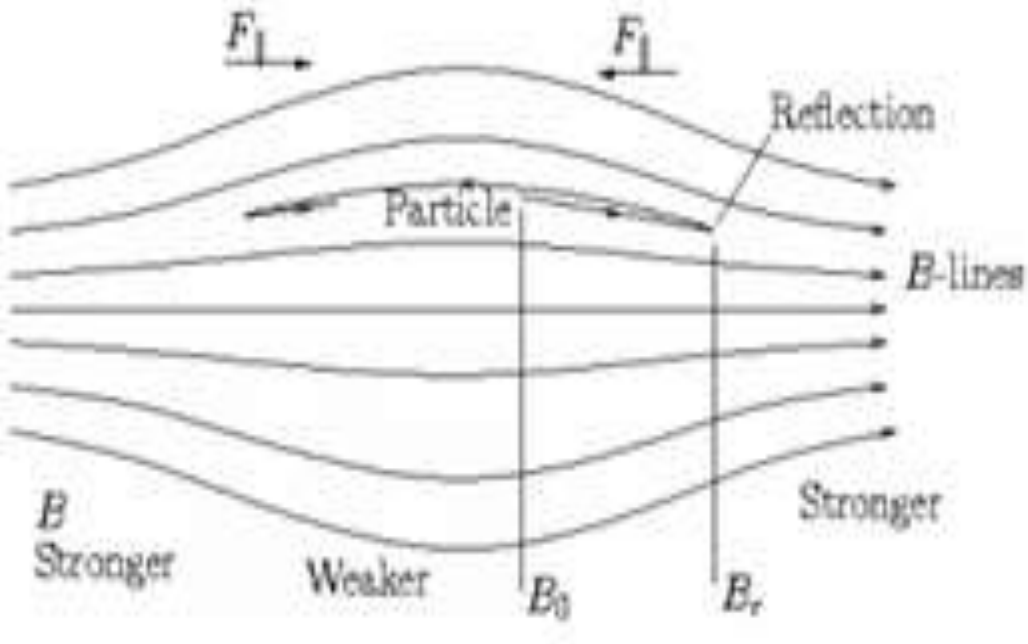


Figure 8. Magnetic Mirror

μ conservation

$$\frac{\frac{1}{2}mv_{\perp 0}^2}{B_0} = \frac{\frac{1}{2}mv_{\perp r}^2}{B_r} \qquad (56)$$

$$v_{\perp 0}^2 + v_{\parallel 0}^2 = \frac{B_r}{B_0} v_{\perp 0}^2 \qquad (57)$$

$$\frac{B_0}{B_r} = \frac{v_{\perp 0}^2}{v_{\perp 0}^2 + v_{\parallel o}^2} \quad (58)$$

Pitch Angle θ

$$\tan\theta = \frac{v_\perp}{v_\parallel} \quad (59)$$

$$\frac{B_0}{B_r} = \frac{v_{\perp 0}^2}{v_{\perp 0}^2 + v_{\parallel 0}^2} = \sin^2\theta_0 \quad (60)$$

So, given a pitch angle θ_0, reflection takes place where $B_0/B_r = \sin^2 \theta_0$.

If θ_0 is too small no reflection can occur.

Critical angle θ_c is obviously

$$\theta_c = \sin^{-1}(B_0/B_1)^{\frac{1}{2}} \quad (61)$$

Loss Cone is all $\theta < \theta_c$.

Importance of Mirror Ratio: $R_m = B_1/B_0$.

Other Features of Mirror Motions

Flux enclosed by gyro orbit is constant.

$$\Phi = \pi r_L^2 B = \frac{\pi m^2 v_\perp^2}{q^2 B^2} B \quad (62)$$

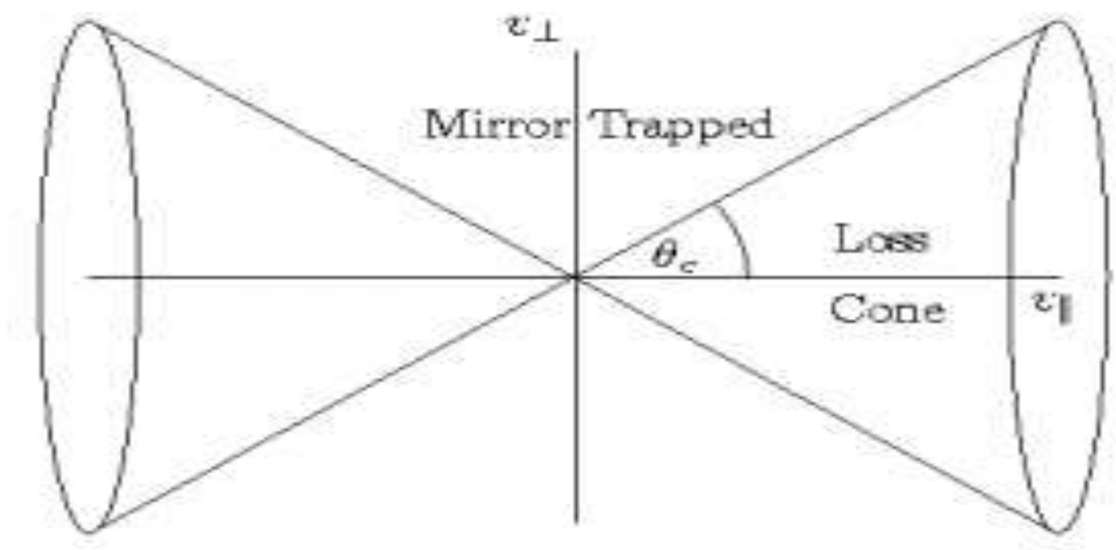


Figure 9. Critical angle θ_c divides velocity space into a loss-cone and a region of mirror-trappin

$$= \frac{2\pi m}{q^2}\frac{\frac{1}{2}mv_\perp^2}{B} \tag{63}$$

$$= \frac{2\pi m}{q^2}\mu = \text{constant.} \tag{64}$$

Note that if B changes ‘suddenly’ μ might not be conserved.

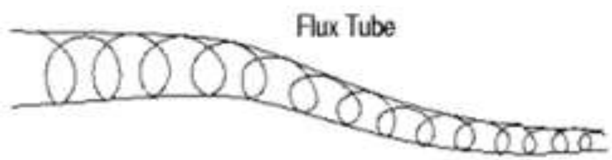


Figure 10. Flux tube described by orbit

Basic requirement

$r_L << B/|\nabla B|$ (65)

Slow variation of B (relative to r_L).