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NAAC Grade 'A'
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Motion of charged particles–L - 03

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Non-Uniform B Field

If B-lines are straight but the magnitude of B varies in space we get orbits that look qualitatively similar to the E $\perp$ B case

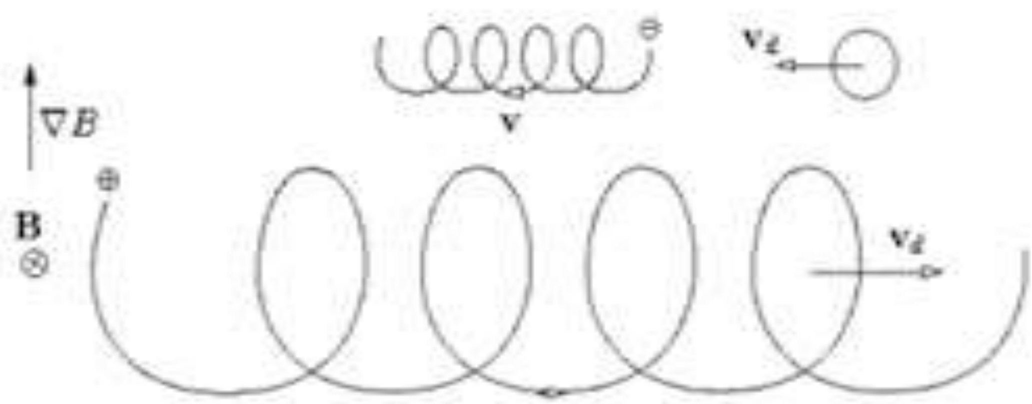

Figure 4 :∇ B drift orbit

Curvature of orbit is greater where B is greater causing loop to be small on that side. Result is a drift perpendicular to both B and ∇B. Notice, though, that electrons and ions go in opposite directions (unlike $E \wedge B$).

Algebra

We try to find a decomposition of the velocity as before into $v = v_d + v_L$ where v_d is constant.

We shall find that this can be done only approximately. Also we must have a simple expression for B. This we get by assuming that the Larmor radius is much smaller than the scale length of B variation i.e.,

$$r_L << B/|\nabla B| \quad (22)$$

in which case we can express the field approximately as the first two terms in a Taylor expression:

$$B = B_0 + (r.\nabla)B \quad (23)$$

Then substituting the decomposed velocity we get:

$$m\frac{d\mathrm{v}}{dt} = m\dot{\mathrm{v}}_L = q(\mathrm{v} \wedge \mathrm{B}) = q[\mathrm{v}_L \wedge \mathrm{B}_0 + \mathrm{v}_d \wedge \mathrm{B}_0 + (\mathrm{v}_L + \mathrm{v}_d) \wedge (\mathrm{r}.\nabla)\mathrm{B}] \quad (24)$$

$e^{-i\Omega t}$

$$\text{or} \quad 0 = \mathbf{v}_d \wedge \mathbf{B}_0 + \mathbf{v}_L \wedge (\mathbf{r}.\nabla)\mathbf{B} + \mathbf{v}_d \wedge (\mathbf{r}.\nabla)\mathbf{B} \qquad (25)$$

Now we shall find that v_d/v_L is also small, like $r|\nabla B|B$. Therefore the last term here is second order but the first two are first order. So we drop the last term.

Now the awkward part is that v_L and r_L are periodic. Substitute for $r = r_0 + r_L$ so

$$0 = v_d \wedge B_0 + v_L \wedge (r_L.\nabla)B + v_L \wedge (r_0.\nabla)B \qquad (26)$$

We now average over a cyclotron period. The last term $\propto e^{-i\Omega t}$ so is it averages to zero:

$$0 = \mathbf{v}_d \wedge \mathbf{B} + \langle \mathbf{v}_L \wedge (\mathbf{r}_L.\nabla)\mathbf{B} \rangle \; . \tag{27}$$

To perform the average use

$$\mathbf{r}_L = (x_L, y_L) \; = \; \frac{v_\perp}{\Omega}\left(\sin \Omega t, \frac{q}{|q|}\cos \Omega t\right) \tag{28}$$

$$\mathbf{v}_L = (\dot{x}_L, \dot{y}_L) \; = \; v_\perp \left(\cos \Omega t, \frac{-q}{|q|}\sin \Omega t\right) \tag{29}$$

So $\quad [v_L \wedge (\mathbf{r}.\nabla)\mathbf{B}]_x \; = \; v_y y \frac{d}{dy} B$ (30)

$$[\mathbf{v}_L \wedge (\mathbf{r}.\nabla)\mathbf{B}]_y \; = \; -v_x y \frac{d}{dy} B \tag{31}$$

(Taking ∇B to be in the y-direction).

Then

$$\langle v_y y\rangle = -\langle \cos\Omega t \sin\Omega t\rangle \frac{v_\perp^2}{\Omega} = 0 \quad (32)$$

$$\langle v_x y\rangle = \frac{q}{|q|}\langle \cos\Omega t \cos\Omega t\rangle \frac{v_\perp^2}{\Omega} = \frac{1}{2}\frac{v_\perp^2}{\Omega}\frac{q}{|q|} \quad (33)$$

So

$$\langle \mathbf{v}_L \wedge (\mathbf{r}.\nabla)\mathbf{B}\rangle = -\frac{q}{|q|}\,\frac{1}{2}\frac{v_\perp^2}{\Omega}\nabla B \quad (34)$$

Substitute in:

$$0 = \mathbf{v}_d \wedge \mathbf{B} - \frac{q}{|q|}\frac{v_\perp^2}{2\Omega}\nabla B \quad (35)$$

and solve as before to get

$$\mathbf{v}_d = \frac{\left(\frac{-1}{|q|}\frac{v_\perp^2}{2\Omega}\,\nabla B\right)\wedge\mathbf{B}}{B^2} = \frac{q}{|q|}\frac{v_\perp^2}{2\Omega}\frac{\mathbf{B}\wedge\nabla B}{B^2} \quad (36)$$

or equivalently

$$\mathbf{v}_d = \frac{1}{q}\frac{mv_\perp^2}{2B}\frac{\mathbf{B}\wedge\nabla B}{B^2} \tag{37}$$

Curvature Drift

When the B field lines are curved and the particle has a velocity $v_{||}$ along the field, another drift occurs.

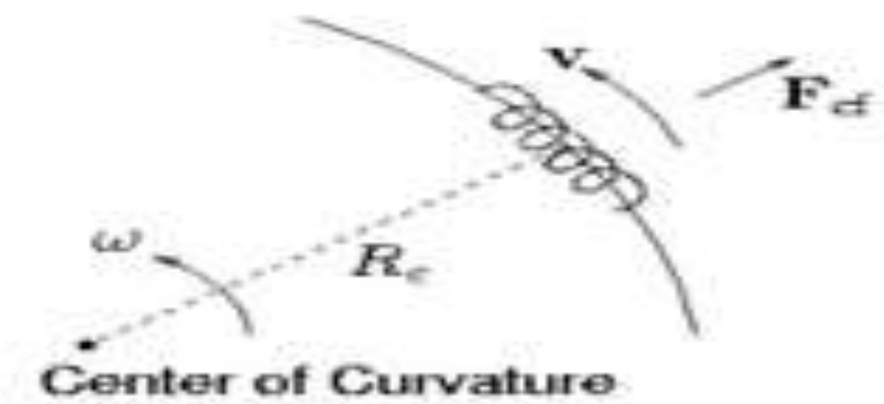

Figure 5 : Curvature and Centrifugal Force

Take |B| constant; radius of curvature R_e.

To 1st order the particle just spirals along the field.

In the frame of the guiding center a force appears because the plasma is rotating about the center of curvature.

This centrifugal force is F_{cf}

$$F_{cf} = m\frac{v_{\parallel}^2}{R_c} \text{ pointing outward} \tag{38}$$

as a vector

$$\mathbf{F}_{cf} = mv_{\parallel}^2\frac{\mathbf{R}_c}{R_c^2} \tag{39}$$

[There is also a coriolis force 2m(ω ∧ v) but this averages to zero over a gyroperiod.] Use the previous formula for a force

$$\mathbf{v}_d = \frac{1}{q}\frac{\mathbf{F}_{cf} \wedge \mathbf{B}}{B^2} = \frac{mv_\parallel^2}{qB^2}\frac{\mathbf{R}_c \wedge \mathbf{B}}{R_c^2} \tag{40}$$

This is the "Curvature Drift".

It is often convenient to have this expressed in terms of the field gradients. So we relate R_c to ∇B etc. as follows:

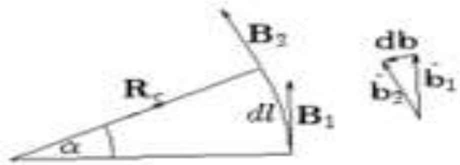

Figure 6. Differential expression of curvature.

(Carets denote unit vectors)

From the diagram

Db= bˆ$_2$− bˆ1= −Rˆ$_c$ a (41)

And

$$d\ell = \propto R_c \quad (42)$$

So

$$\frac{d\mathbf{b}}{dl} = -\frac{\hat{\mathbf{R}}_c}{R_c} = -\frac{\mathbf{R}_c}{R_c^2} \quad (43)$$

But (by definition)

$$\frac{d\mathbf{b}}{dl} = (\hat{\mathbf{B}}.\nabla)\hat{\mathbf{b}} \quad (44)$$

So the curvature drift can be written

$$\mathbf{v}_d = \frac{mv_\parallel^2}{q}\frac{\mathbf{R}_c}{R_c^2} \wedge \frac{\mathbf{B}}{B^2} = \frac{mv_\parallel^2}{q}\frac{\mathbf{B} \wedge (\hat{\mathbf{b}}.\nabla)\hat{\mathbf{b}}}{B^2} \qquad (45)$$