



Langat Singh College, Muzaffarpur

NAAC Grade 'A'

Under B. R. A. Bihar University, Muzaffarpur

Motion of charged particles – L - 02

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Direction of rotation is as indicated opposite for opposite sign of charge

- Ions rotate anticlockwise. Electrons clockwise about the magnetic field.
- The current carried by the plasma always is in such a direction as to reduce the magnetic field. This is the property of a magnetic material which is “Diamagnetic”.
- When is nonzero the total motion is along a helix.

Uniform B and nonzero E

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \quad (12)$$

Parallel motion: Before, when $E = 0$ this was $v = \text{const.}$ Now it is clearly

$$\dot{v}_{\parallel} = \frac{qE_{\parallel}}{m} \quad (13)$$

Constant acceleration along the field.

Perpendicular Motion

Qualitatively:

Speed of positive particle is greater at top than bottom so radius of curvature is greater.

Result is that guiding center moves perpendicular to both E and B . It 'drifts' across the field.

Algebraically: It is clear that if we can find a constant velocity that satisfies

$$\mathbf{E} + \mathbf{V}_d \wedge \mathbf{B} = 0 \quad (14)$$

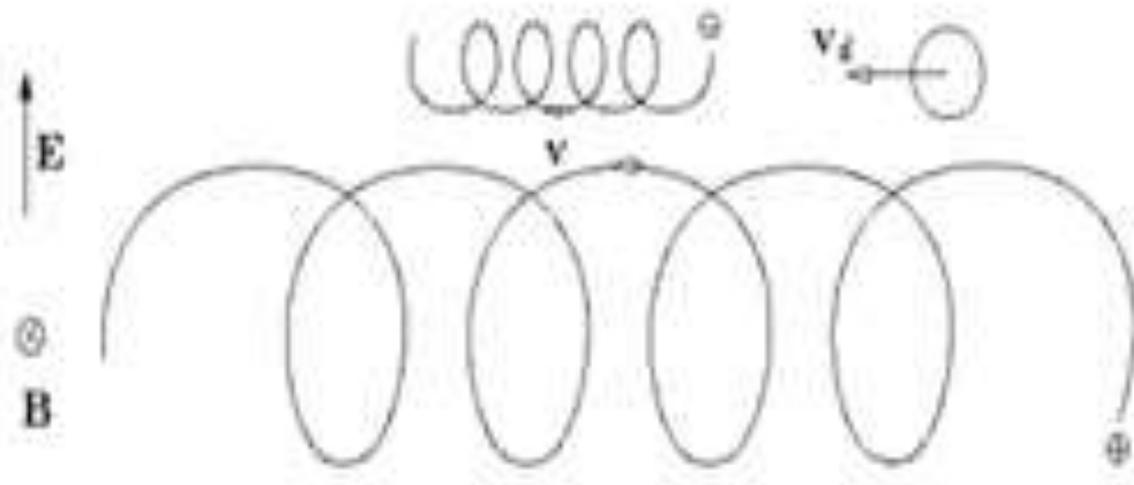


Figure 3 : $\mathbf{E} \wedge \mathbf{B}$ drift orbit

then the sum of this drift velocity plus the velocity

$$\mathbf{v}_L = \frac{d}{dt} [\mathbf{r}_L e^{i\Omega(t-t_0)}] \quad (15)$$

- which we calculated for the $E = 0$ gyration will satisfy the equation of motion. Take $E \wedge B$ the above equation:

$$0 = E \wedge B + (\mathbf{v}_d \wedge B) \wedge B = E \wedge B + (u_d \cdot B) B - B^2 v_d \quad (16)$$

so that

$$\mathbf{v}_d = \frac{\mathbf{E} \wedge \mathbf{B}}{B^2} \quad (17)$$

does satisfy it.

Hence the full solution is

$$\mathbf{v} = \underbrace{v_{\parallel}}_{\text{parallel}} + \underbrace{v_d}_{\text{cross-field drift}} + \underbrace{v_L}_{\text{Gyration}} \quad (18)$$

Where

$$\dot{v}_{\parallel} = \frac{qE_{\parallel}}{m} \quad (19)$$

(Eq 17) is the “ $E \times B$ drift” of the gyrocenter.

Comments on $E \times B$ drift:

1. It is independent of the properties of the drifting particle (q , m , v , whatever).
2. Hence it is in the same direction for electrons and ions.

3. Underlying physics for this is that in the frame moving at the $\mathbf{E} \times \mathbf{B}$ drift $\mathbf{E} = 0$. We have ‘transformed away’ the electric field.
4. Formula given above is exact except for the fact that relativistic effects have been ignored. They would be important if $\sim c$.

Drift due to Gravity or other Forces

Suppose particle is subject to some other force, such as gravity. Write it \mathbf{F} so that

$$m\dot{\mathbf{v}} = \mathbf{F} + q \mathbf{v} \wedge \mathbf{B} = q \left(\frac{1}{q} \mathbf{F} + \mathbf{v} \wedge \mathbf{B} \right) \quad (20)$$

This is just like the Electric field case except with F/q replacing E .

The drift is therefore

$$\mathbf{v}_d = \frac{1}{q} \frac{\mathbf{F} \wedge \mathbf{B}}{B^2} \quad (21)$$

In this case, if force on electrons and ions is same, they drift in opposite directions. This general formula can be used to get the drift velocity in some other cases of interest .