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NAAC Grade 'A'
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Motion of charged particles– L - 01

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Motion of charged particles in fields

Plasmas are complicated because motions of electrons and ions are determined by the electric and magnetic fields but also change the fields by the currents they carry .

Equation of motion

- m = q(E + v $\wedge$ B) (1)
- Lorentz force (F) = m = q(E + v $\wedge$ B)
- q = charge , E = Electric field , B = Magnetic field , v = velocity
- v$\wedge$ B = v $\perp$ B = v x B = velocity perpendicular to magnetic field
- **Uniform B field, E = 0.**
- $m\dot{v}$ = qv Λ B (2)

Qualitatively

- In the plane perpendicular to B: Acceleration is perpendicular to v so particle moves in a circle whose radius r_L is such as to satisfy

$$m r_L \Omega^2 = m \frac{v_\perp^2}{r_L} = |q| v_\perp B \qquad (3)$$

Ω is the angular (velocity) frequency

1st equality shows $\Omega^2 = v_\perp^2 / r_L^2 \quad (r_L = v_\perp / \Omega)$

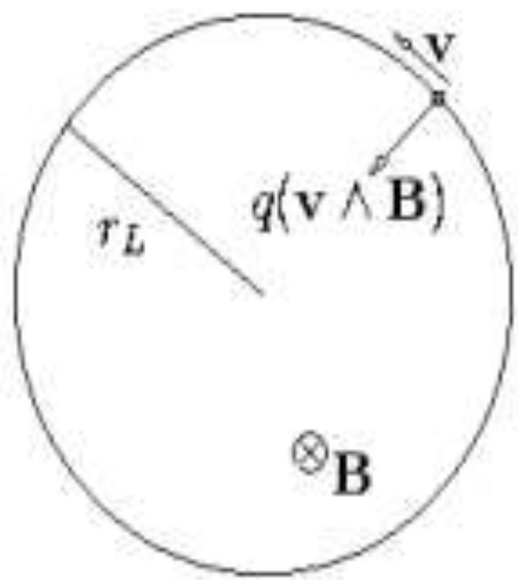

Figure 1: Circular orbit in uniform magnetic field.

Hence second gives $m\frac{v_\perp}{\Omega}\Omega^2 = |q|v_\perp B$

$$\text{i.e.} \quad \Omega = \frac{|q|B}{m} \tag{4}$$

Particle moves in a circular orbit with

angular velocity $\Omega = \dfrac{|q|B}{m}$ the "Cyclotron Frequency" (5)

and radius $r_l = \dfrac{v_\perp}{\Omega}$ the "Larmor Radius. (6)

By Vector Algebra

- Particle Energy is constant. Proof : take v. Eq. of motion then

$$m\mathbf{v}.\dot{\mathbf{v}} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = q\mathbf{v}.(\mathbf{v} \wedge \mathbf{B}) = 0. \qquad (7)$$

- Parallel and Perpendicular motions separate. = constant because acceleration (∝ v ∧ B) is perpendicular to B Perpendicular Dynamics:

Take B in zˆ direction and write components

$$m\dot{v}_x = qv_yB,\ m\dot{u}_y = -qu_xB \tag{8}$$

Hence

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x = -\Omega^2 v_x \tag{9}$$

Solution: = $v_\perp \cos \Omega t$ (choose zero of time)

- Substitute back:

$$v_y = \frac{m}{qB}\dot{v}_x = -\frac{|q|}{q}v_\perp \sin \Omega t \tag{10}$$

Integrate:

$$x = x_0 + \frac{v_\perp}{\Omega}\sin \Omega t \quad , \quad y = y_0 + \frac{q}{|q|}\frac{v_\perp}{\Omega} \cos \Omega t \tag{11}$$

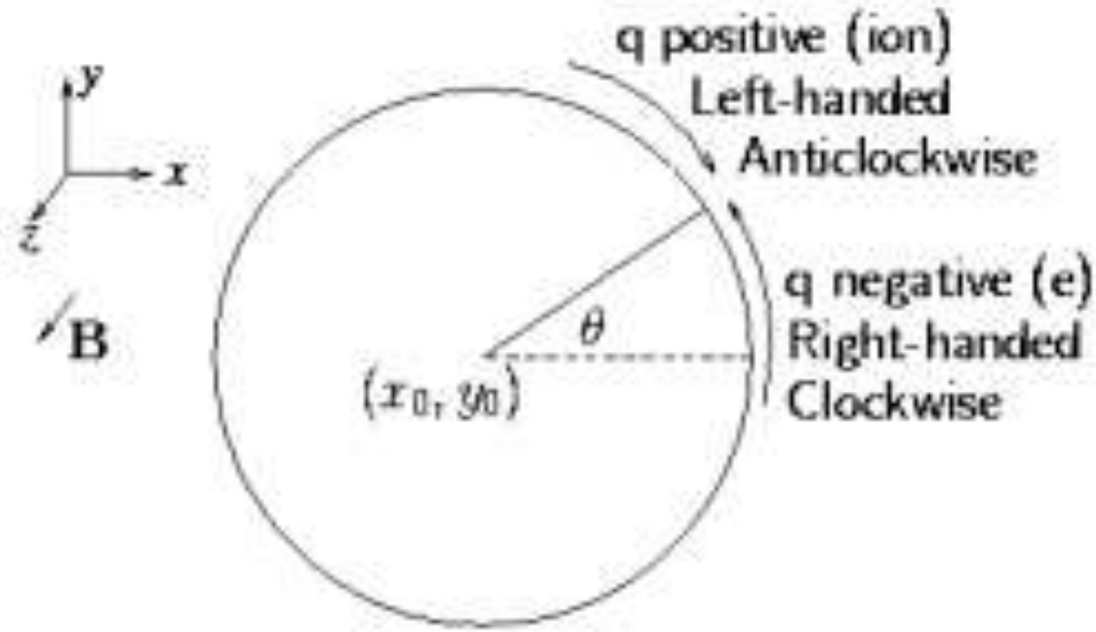

Figure 2: Gyro center (x_0, y_0) and orbit

This is the equation of a circle with center r_0 = (x_0, y_0) and radius = Ω:

Gyro Radius. [Angle is θ = Ωt]