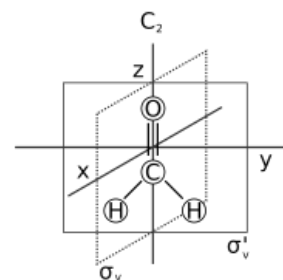


# Molecular symmetry

**Molecular symmetry** in chemistry describes the symmetry present in molecules and the classification of molecules according to their symmetry. Molecular symmetry is a fundamental concept in chemistry, as it can be used to predict or explain many of a molecule's chemical properties, such as its [dipole moment](#) and its allowed [spectroscopic transitions](#). To do this it is necessary to classify the states of the molecule using the [irreducible representations](#) from the [character table](#) of the symmetry group of the molecule. Many university level textbooks on [physical chemistry](#), [quantum chemistry](#), [spectroscopy](#) and [inorganic chemistry](#) devote a chapter to symmetry.<sup>[1][2][3][4][5][6]</sup>

The framework for the study of molecular symmetry is provided by [group theory](#), and in particular [irreducible representation theory](#). Symmetry is useful in the study of [molecular orbitals](#), with applications such as the [Hückel method](#), [ligand field theory](#), and the [Woodward-Hoffmann rules](#). Another framework on a larger scale is the use of [crystal systems](#) to describe [crystallographic symmetry](#) in bulk materials.

Many techniques for the practical assessment of molecular symmetry exist, including [X-ray crystallography](#) and various forms of [spectroscopy](#). [Spectroscopic notation](#) is based on symmetry considerations.



Symmetry elements of formaldehyde.  $C_2$  is a two-fold rotation axis.  $\sigma_v$  and  $\sigma_v'$  are two non-equivalent reflection planes.

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## Symmetry concepts

The study of symmetry in molecules makes use of [group theory](#).

### Elements

The point group symmetry of a molecule can be described by 5 types of [symmetry element](#).

- Symmetry axis:** an axis around which a [rotation](#) by  $\frac{360^\circ}{n}$  results in a molecule indistinguishable from the original. This is also called an *n*-fold **rotational axis** and abbreviated  $C_n$ . Examples are the  $C_2$  axis in [water](#) and the  $C_3$  axis in [ammonia](#). A molecule can have more than one symmetry axis; the one with the highest *n* is called the **principal axis**, and by convention is aligned with the z-axis in a [Cartesian coordinate system](#).
- Plane of symmetry:** a plane of reflection through which an identical copy of the original molecule is generated. This is also called a [mirror plane](#) and abbreviated  $\sigma$  (sigma = Greek "s", from the German 'Spiegel' meaning mirror).<sup>[7]</sup> Water has two of them: one in the plane of the molecule itself and one [perpendicular](#) to it. A symmetry plane [parallel](#) with the principal axis is dubbed *vertical* ( $\sigma_v$ ) and one [perpendicular](#) to it *horizontal* ( $\sigma_h$ ). A third type of symmetry plane exists: If a vertical symmetry plane additionally bisects the angle between two 2-fold rotation axes perpendicular to the principal axis, the plane is dubbed [dihedral](#) ( $\sigma_d$ ). A symmetry plane can also be identified by its Cartesian orientation, e.g., (xz) or (yz).
- Center of symmetry** or **inversion center**, abbreviated *i*. A molecule has a center of symmetry when, for any atom in the molecule, an identical atom exists diametrically opposite this center an equal distance from it. In other words, a molecule has

a center of symmetry when the points (x,y,z) and (-x,-y,-z) correspond to identical objects. For example, if there is an oxygen atom in some point (x,y,z), then there is an oxygen atom in the point (-x,-y,-z). There may or may not be an atom at the inversion center itself. Examples are xenon tetrafluoride where the inversion center is at the Xe atom, and benzene (C<sub>6</sub>H<sub>6</sub>) where the inversion center is at the center of the ring.

- **Rotation-reflection axis:** an axis around which a rotation by  $\frac{360^\circ}{n}$ , followed by a reflection in a plane perpendicular to it, leaves the molecule unchanged. Also called an *n*-fold **improper rotation axis**, it is abbreviated S<sub>n</sub>. Examples are present in tetrahedral silicon tetrafluoride, with three S<sub>4</sub> axes, and the staggered conformation of ethane with one S<sub>6</sub> axis. An S<sub>1</sub> axis corresponds to a mirror plane σ and an S<sub>2</sub> axis is an inversion center *i*. A molecule which has no S<sub>n</sub> axis for any value of n is a chiral molecule.

- **Identity**, abbreviated to E, from the German 'Einheit' meaning unity.<sup>[8]</sup> This symmetry element simply consists of no change: every molecule has this element. While this element seems physically trivial, it must be included in the list of symmetry elements so that they form a mathematical group, whose definition requires inclusion of the identity element. It is so called because it is analogous to multiplying by one (unity). In other words, E is a property that any object needs to have regardless of its symmetry properties.<sup>[9]</sup>

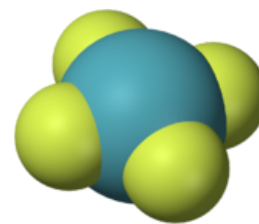
Examples of the relationship between chirality and symmetry

Rotational axis (C <sub>n</sub> )	Improper rotational elements (S <sub>n</sub> )		
	Chiral no S <sub>n</sub>	Achiral mirror plane S <sub>1</sub> = σ	Achiral inversion centre S <sub>2</sub> = <i>i</i>
C <sub>1</sub>			
C <sub>2</sub>			

## Operations

The five symmetry elements have associated with them five types of **symmetry operation**, which leave the molecule in a state indistinguishable from the starting state. They are sometimes distinguished from symmetry elements by a caret or circumflex. Thus,  $\hat{C}_n$  is the rotation of a molecule around an axis and  $\hat{E}$  is the identity operation. A symmetry element can have more than one symmetry operation associated with it. For example, the C<sub>4</sub> axis of the square xenon tetrafluoride (XeF<sub>4</sub>) molecule is associated with two  $\hat{C}_4$  rotations (90°) in opposite directions and a  $\hat{C}_2$  rotation (180°). Since  $\hat{C}_1$  is equivalent to  $\hat{E}$ ,  $\hat{S}_1$  to σ and  $\hat{S}_2$  to  $\hat{i}$ , all symmetry operations can be classified as either proper or improper rotations.

For linear molecules, either clockwise or counterclockwise rotation about the molecular axis by any angle Φ is a symmetry operation.



XeF<sub>4</sub>, with square planar geometry, has 1 C<sub>4</sub> axis and 4 C<sub>2</sub> axes orthogonal to C<sub>4</sub>. These five axes plus the mirror plane perpendicular to the C<sub>4</sub> axis define the D<sub>4h</sub> symmetry group of the molecule.

## Symmetry groups

### Groups

The symmetry operations of a molecule (or other object) form a group. In mathematics, a group is a set with a binary operation that satisfies the four properties listed below.

In a **symmetry group**, the group elements are the symmetry operations (not the symmetry elements), and the binary combination consists of applying first one symmetry operation and then the other. An example is the sequence of a C<sub>4</sub> rotation about the z-axis and a reflection in the xy-plane, denoted σ(xy)C<sub>4</sub>. By convention the order of operations is from right to left.

A symmetry group obeys the defining properties of any group.

#### (1) closure property:

For every pair of elements *x* and *y* in *G*, the *product* *x*\**y* is also in *G*.  
( in symbols, for every two elements *x*, *y* ∈ *G*, *x*\**y* is also in *G* ).

This means that the group is *closed* so that combining two elements produces no new elements. Symmetry operations have this property because a sequence of two operations will produce a third state indistinguishable from the second and therefore from the first, so that the net effect on the molecule is still a symmetry operation.

#### (2) Associative property:

For every *x* and *y* and *z* in *G*, both (*x*\**y*)\**z* and *x*\*(*y*\**z*) result with the same element in *G*.  
( in symbols, (*x*\**y*)\**z* = *x*\*(*y*\**z*) for every *x*, *y*, and *z* ∈ *G* )

#### (3) existence of identity property:

There must be an element ( say *e* ) in *G* such that product any element of *G* with *e* make no change to the element.  
( in symbols, *x*\**e*=*e*\**x*= *x* for every *x* ∈ *G* )

#### (4) existence of inverse element:

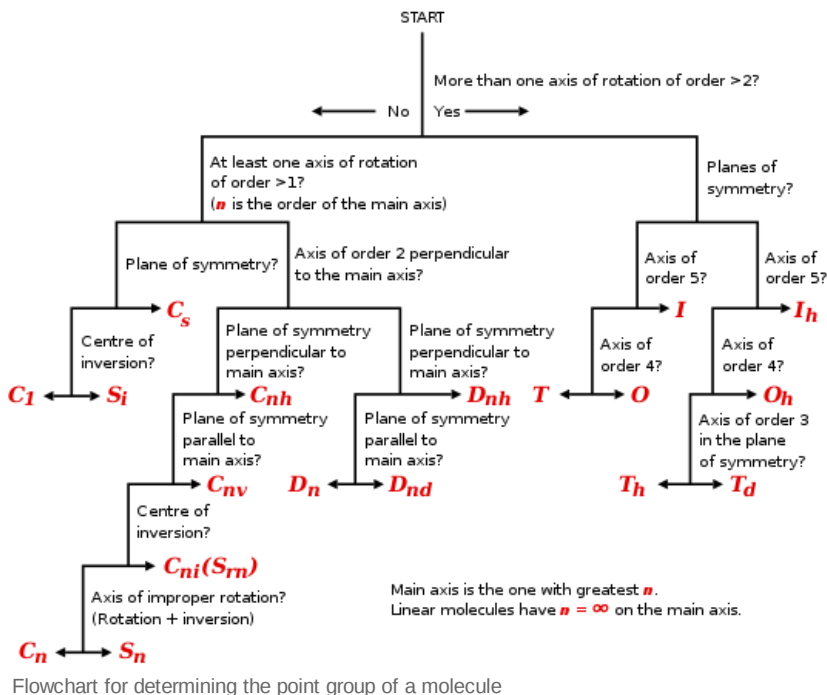
For each element ( $x$ ) in  $G$ , there must be an element  $y$  in  $G$  such that product of  $x$  and  $y$  is the identity element  $e$ .  
(in symbols, for each  $x \in G$  there is a  $y \in G$  such that  $x*y=y*x=e$  for every  $x \in G$ )

The *order* of a group is the number of elements in the group. For groups of small orders, the group properties can be easily verified by considering its composition table, a table whose rows and columns correspond to elements of the group and whose entries correspond to their products.

## Point groups and permutation-inversion groups

The successive application (or *composition*) of one or more symmetry operations of a molecule has an effect equivalent to that of some single symmetry operation of the molecule. For example, a  $C_2$  rotation followed by a  $\sigma_v$  reflection is seen to be a  $\sigma_v$  symmetry operation:  $\sigma_v * C_2 = \sigma_v$ . ("Operation A followed by B to form C" is written  $BA = C$ ).<sup>[9]</sup> Moreover, the set of all symmetry operations (including this composition operation) obeys all the properties of a group, given above. So  $(S,*)$  is a group, where  $S$  is the set of all symmetry operations of some molecule, and  $*$  denotes the composition (repeated application) of symmetry operations.

This group is called the point group of that molecule, because the set of symmetry operations leave at least one point fixed (though for some symmetries an entire axis or an entire plane remains fixed). In other words, a point group is a group that summarizes all symmetry operations that all molecules in that category have.<sup>[9]</sup> The symmetry of a crystal, by contrast, is described by a space group of symmetry operations, which includes translations in space.



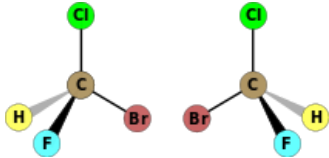
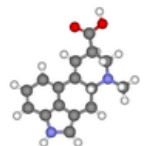
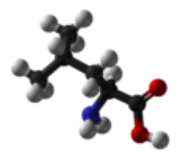
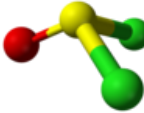


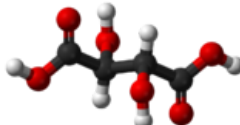
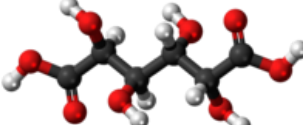

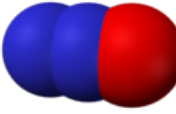



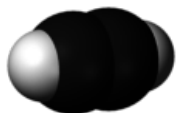
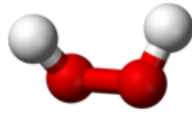
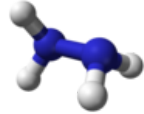
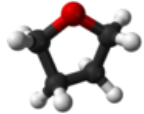
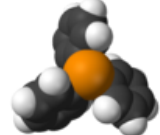
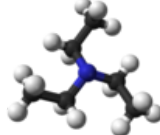
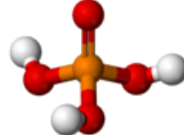
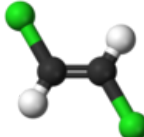
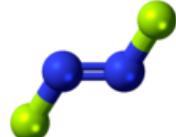
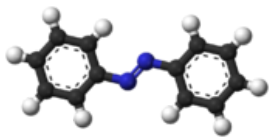

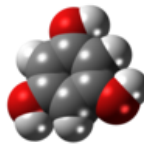

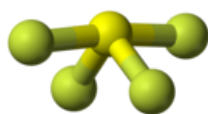
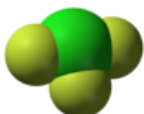
One can determine the symmetry operations of the point group for a particular molecule by considering the geometrical symmetry of its molecular model. However, when one USES a point group to classify molecular states, the operations in it are not to be interpreted in the same way. Instead the operations are interpreted as rotating and/or reflecting the vibronic (vibration-electronic) coordinates<sup>[10]</sup> and these operations commute with the vibronic Hamiltonian. They are "symmetry operations" for that vibronic Hamiltonian. The point group is used to classify by symmetry the vibronic eigenstates. The symmetry classification of the rotational levels, the eigenstates of the full (rotation-vibration-electronic) Hamiltonian, requires the use of the appropriate permutation-inversion group as introduced by Longuet-Higgins.<sup>[11]</sup>


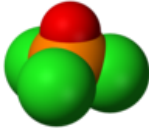
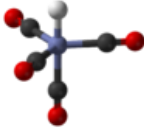
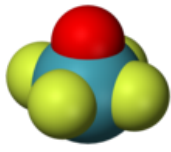
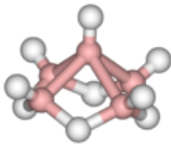

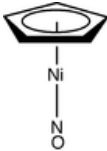
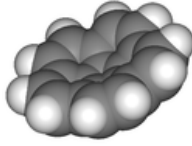
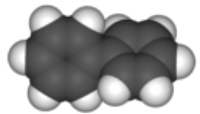
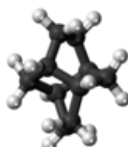
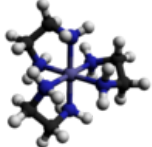
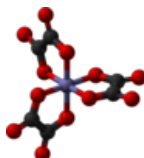


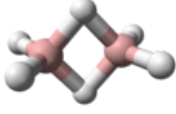
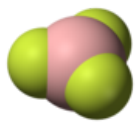


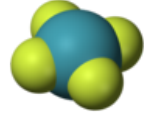
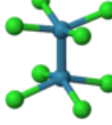

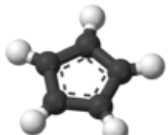





## Examples of point groups


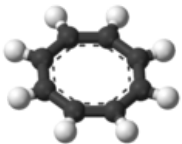

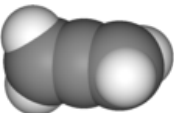
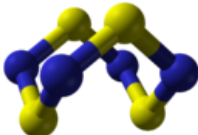
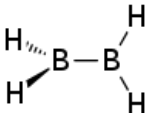
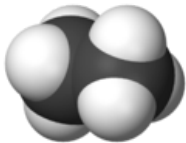
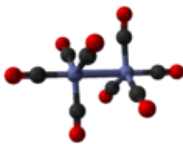

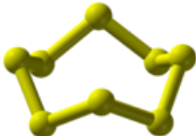
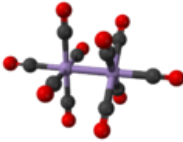

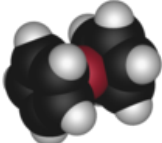
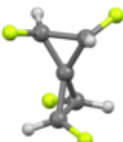

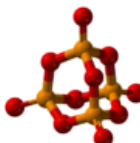
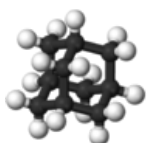
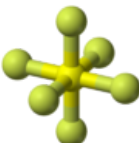


Assigning each molecule a point group classifies molecules into categories with similar symmetry properties. For example,  $PCl_3$ ,  $POF_3$ ,  $XeO_3$ , and  $NH_3$  all share identical symmetry operations.<sup>[12]</sup> They all can undergo the identity operation  $E$ , two different  $C_3$  rotation operations, and three different  $\sigma_v$  plane reflections without altering their identities, so they are placed in one point group,  $C_{3v}$ , with order 6.<sup>[13]</sup> Similarly, water ( $H_2O$ ) and hydrogen sulfide ( $H_2S$ ) also share identical symmetry operations. They both undergo the identity operation  $E$ , one  $C_2$  rotation, and two  $\sigma_v$  reflections without altering their identities, so they are both placed in one point group,  $C_{2v}$ , with order 4.<sup>[14]</sup> This classification system helps scientists to study molecules more efficiently, since chemically related molecules in the same point group tend to exhibit similar bonding schemes, molecular bonding diagrams, and spectroscopic properties.<sup>[9]</sup>

## Common point groups


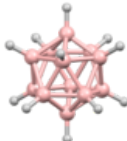

The following table contains a list of point groups labelled using the Schoenflies notation, which is common in chemistry and molecular spectroscopy. The description of structure includes common shapes of molecules, which can be explained by the VSEPR model.

Point group	Symmetry operations <sup>[15]</sup>	Simple description of typical geometry	Example 1	Example 2	Example 3
$C_1$	E	no symmetry, chiral	 bromochlorofluoromethane (both enantiomers shown)	 lysergic acid	 L-leucine and most other $\alpha$ -amino acids except glycine
$C_s$	$E \sigma_h$	mirror plane, no other symmetry	 thionyl chloride	 hypochlorous acid	 chloriodomethane
$C_i$	$E i$	inversion center	 meso-tartaric acid	 mucic acid (meso-galactaric acid)	(S,R) 1,2-dibromo-1,2-dichloroethane ( <i>anti</i> conformer)
$C_{\infty v}$	$E 2C_{\infty}^{\Phi} \infty \sigma_v$	linear	 hydrogen fluoride (and all other heteronuclear diatomic molecules)	 nitrous oxide (dinitrogen monoxide)	 hydrocyanic acid (hydrogen cyanide)
$D_{\infty h}$	$E 2C_{\infty}^{\Phi} \infty \sigma_i i 2S_{\infty}^{\Phi} \infty C_2$	linear with inversion center	 oxygen (and all other homonuclear diatomic molecules)	 carbon dioxide	 acetylene (ethyne)
$C_2$	$E C_2$	"open book geometry", chiral	 hydrogen peroxide	 hydrazine	 tetrahydrofuran (twist conformation)
$C_3$	$E C_3$	propeller, chiral	 triphenylphosphine	 triethylamine	 phosphoric acid
$C_{2h}$	$E C_2 i \sigma_h$	planar with inversion center, no vertical plane	 trans-1,2-dichloroethylene	 trans-dinitrogen difluoride	 trans-azobenzene
$C_{3h}$	$E C_3 C_3^2 \sigma_h S_3 S_3^5$	propeller	 boric acid	 phloroglucinol (1,3,5-trihydroxybenzene)	
$C_{2v}$	$E C_2 \sigma_v(xz) \sigma_v'(yz)$	angular ( $H_2O$ ) or see-saw ( $SF_4$ ) or T-shape ( $ClF_3$ )	 water	 sulfur tetrafluoride	 chlorine trifluoride

$C_{3v}$	$E\ 2C_3\ 3\sigma_v$	trigonal pyramidal	 non-inverting ammonia	 phosphorus oxychloride	 cobalt tetracarbonyl hydride, $HCo(CO)_4$
$C_{4v}$	$E\ 2C_4\ C_2\ 2\sigma_v\ 2\sigma_d$	square pyramidal	 xenon oxytetrafluoride	 pentaborane(9), $B_5H_9$	 nitroprusside anion $[Fe(CN)_5(NO)]^{2-}$
$C_{5v}$	$E\ 2C_5\ 2C_5^2\ 5\sigma_v$	'milking stool' complex	 $Ni(C_5H_5)(NO)$	 corannulene	
$D_2$	$E\ C_2(x)\ C_2(y)\ C_2(z)$	twist, chiral	 biphenyl (skew conformation)	 twistane ( $C_{10}H_{16}$ )	cyclohexane twist conformation
$D_3$	$E\ C_3(z)\ 3C_2$	triple helix, chiral	 Tris(ethylenediamine)cobalt(III) cation	 tris(oxalato)iron(III) anion	
$D_{2h}$	$E\ C_2(z)\ C_2(y)\ C_2(x)\ i\ \sigma(xy)\ \sigma(xz)\ \sigma(yz)$	planar with inversion center, vertical plane	 ethylene	 pyrazine	 diborane
$D_{3h}$	$E\ 2C_3\ 3C_2\ \sigma_h\ 2S_3\ 3\sigma_v$	trigonal planar or trigonal bipyramidal	 boron trifluoride	 phosphorus pentachloride	 cyclopropane
$D_{4h}$	$E\ 2C_4\ C_2\ 2C_2'\ 2C_2''\ i\ 2S_4\ \sigma_h\ 2\sigma_v\ 2\sigma_d$	square planar	 xenon tetrafluoride	 octachlorodimolybdate(II) anion	 $Trans-[Co^{III}(NH_3)_4Cl_2]^+$ (excluding H atoms)
$D_{5h}$	$E\ 2C_5\ 2C_5^2\ 5C_2\ \sigma_h\ 2S_5\ 2S_5^3\ 5\sigma_v$	pentagonal	 cyclopentadienyl anion	 ruthenocene	 $C_{70}$
$D_{6h}$	$E\ 2C_6\ 2C_3\ C_2\ 3C_2'\ 3C_2''\ i\ 2S_3\ 2S_6\ \sigma_h\ 3\sigma_d\ 3\sigma_v$	hexagonal	 benzene	 bis(benzene)chromium	 coronene ( $C_{24}H_{12}$ )

D <sub>7h</sub>	E C <sub>7</sub> S <sub>7</sub> 7C <sub>2</sub> σ <sub>h</sub> 7σ <sub>v</sub>	heptagonal			
			<u>tropylium</u> (C <sub>7</sub> H <sub>7</sub> <sup>+</sup> ) cation		
D <sub>8h</sub>	E C <sub>8</sub> C <sub>4</sub> C <sub>2</sub> S <sub>8</sub> i 8C <sub>2</sub> σ <sub>h</sub> 4σ <sub>v</sub> 4σ <sub>d</sub>	octagonal			
			<u>cyclooctatetraene</u> (C <sub>8</sub> H <sub>8</sub> <sup>2-</sup> ) anion	<u>uranocene</u>	
D <sub>2d</sub>	E 2S <sub>4</sub> C <sub>2</sub> 2C <sub>2</sub> ' 2σ <sub>d</sub>	90° twist			
			<u>allene</u>	<u>tetrasulfur tetranitride</u>	<u>diborane(4)</u> (excited state)
D <sub>3d</sub>	E 2C <sub>3</sub> 3C <sub>2</sub> i 2S <sub>6</sub> 3σ <sub>d</sub>	60° twist			
			<u>ethane</u> (staggered rotamer)	<u>dicobalt octacarbonyl</u> (non-bridged isomer)	<u>cyclohexane chair conformation</u>
D <sub>4d</sub>	E 2S <sub>8</sub> 2C <sub>4</sub> 2S <sub>8</sub> <sup>3</sup> C <sub>2</sub> 4C <sub>2</sub> ' 4σ <sub>d</sub>	45° twist			
			<u>sulfur</u> (crown conformation of S <sub>8</sub> )	<u>dimanganese decacarbonyl</u> (staggered rotamer)	<u>octafluoroxenate ion</u> (idealized geometry)
D <sub>5d</sub>	E 2C <sub>5</sub> 2C <sub>5</sub> <sup>2</sup> 5C <sub>2</sub> i 2S <sub>10</sub> <sup>3</sup> 2S <sub>10</sub> 5σ <sub>d</sub>	36° twist			
			<u>ferrocene</u> (staggered rotamer)		
S <sub>4</sub>	E 2S <sub>4</sub> C <sub>2</sub>				
			<u>1,2,3,4-tetrafluorospiro[3.3]heptane</u> <sup>[16]</sup>		
T <sub>d</sub>	E 8C <sub>3</sub> 3C <sub>2</sub> 6S <sub>4</sub> 6σ <sub>d</sub>	<u>tetrahedral</u>			
			<u>methane</u>	<u>phosphorus pentoxide</u>	<u>adamantane</u>
T <sub>h</sub>	E 4C <sub>3</sub> 4C <sub>3</sub> <sup>2</sup> i 3C <sub>2</sub> 4S <sub>6</sub> 4S <sub>6</sub> <sup>5</sup> 3σ <sub>h</sub>	<u>pyritohedron</u>			
O <sub>h</sub>	E 8C <sub>3</sub> 6C <sub>2</sub> 6C <sub>4</sub> 3C <sub>2</sub> i 6S <sub>4</sub> 8S <sub>6</sub> 3σ <sub>h</sub> 6σ <sub>d</sub>	<u>octahedral</u> or <u>cubic</u>			
			<u>sulfur hexafluoride</u>	<u>molybdenum hexacarbonyl</u>	<u>cubane</u>
I <sub>h</sub>	E 12C <sub>5</sub> 12C <sub>5</sub> <sup>2</sup> 20C <sub>3</sub> 15C <sub>2</sub> i	<u>icosahedral</u> or <u>dodecahedral</u>			



12S <sub>10</sub> 12S <sub>10</sub> <sup>3</sup> 20S <sub>6</sub> 15σ			
	Buckminsterfullerene	dodecaborate anion	dodecahedrane

## Representations

The symmetry operations can be represented in many ways. A convenient representation is by matrices. For any vector representing a point in Cartesian coordinates, left-multiplying it gives the new location of the point transformed by the symmetry operation. Composition of operations corresponds to matrix multiplication. Within a point group, a multiplication of the matrices of two symmetry operations leads to a matrix of another symmetry operation in the same point group.<sup>[9]</sup> For instance, in the C<sub>2v</sub> example this is:

$$\underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C_2} \times \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\sigma_v} = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\sigma'_v}$$

Although an infinite number of such representations exist, the irreducible representations (or "irreps") of the group are commonly used, as all other representations of the group can be described as a linear combination of the irreducible representations.

## Character tables

For each point group, a **character table** summarizes information on its symmetry operations and on its irreducible representations. As there are always equal numbers of irreducible representations and classes of symmetry operations, the tables are square.

The table itself consists of **characters** that represent how a particular irreducible representation transforms when a particular symmetry operation is applied. Any symmetry operation in a molecule's point group acting on the molecule itself will leave it unchanged. But, for acting on a general entity, such as a vector or an orbital, this need not be the case. The vector could change sign or direction, and the orbital could change type. For simple point groups, the values are either 1 or -1: 1 means that the sign or phase (of the vector or orbital) is unchanged by the symmetry operation (*symmetric*) and -1 denotes a sign change (*asymmetric*).

The representations are labeled according to a set of conventions:

- A, when rotation around the principal axis is symmetrical
- B, when rotation around the principal axis is asymmetrical
- E and T are doubly and triply degenerate representations, respectively
- when the point group has an inversion center, the subscript g (German: *gerade* or even) signals no change in sign, and the subscript u (*ungerade* or uneven) a change in sign, with respect to inversion.
- with point groups C<sub>∞v</sub> and D<sub>∞h</sub> the symbols are borrowed from angular momentum description: Σ, Π, Δ.

The tables also capture information about how the Cartesian basis vectors, rotations about them, and quadratic functions of them transform by the symmetry operations of the group, by noting which irreducible representation transforms in the same way. These indications are conventionally on the righthand side of the tables. This information is useful because chemically important orbitals (in particular *p* and *d* orbitals) have the same symmetries as these entities.

The character table for the C<sub>2v</sub> symmetry point group is given below:

C <sub>2v</sub>	E	C <sub>2</sub>	σ <sub>v</sub> (xz)	σ <sub>v'</sub> (yz)		
A <sub>1</sub>	1	1	1	1	z	x <sup>2</sup> , y <sup>2</sup> , z <sup>2</sup>
A <sub>2</sub>	1	1	-1	-1	R <sub>z</sub>	xy
B <sub>1</sub>	1	-1	1	-1	x, R <sub>y</sub>	xz
B <sub>2</sub>	1	-1	-1	1	y, R <sub>x</sub>	yz

Consider the example of water (H<sub>2</sub>O), which has the C<sub>2v</sub> symmetry described above. The 2p<sub>x</sub> orbital of oxygen has B<sub>1</sub> symmetry as in the fourth row of the character table above, with x in the sixth column). It is oriented perpendicular to the plane of the molecule and switches sign with a C<sub>2</sub> and a σ<sub>v'</sub>(yz) operation, but remains unchanged with the other two operations (obviously, the character for the identity operation is always +1). This orbital's character set is thus {1, -1, 1, -1}, corresponding to the B<sub>1</sub> irreducible representation. Likewise, the 2p<sub>z</sub> orbital is seen to have the symmetry of the A<sub>1</sub> irreducible representation (*i.e.*: none of the symmetry operations change it), 2p<sub>y</sub> B<sub>2</sub>, and the 3d<sub>xy</sub> orbital A<sub>2</sub>. These assignments and others are noted in the rightmost two columns of the table.

## Historical background

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Hans Bethe used characters of point group operations in his study of [ligand field theory](#) in 1929, and Eugene Wigner used group theory to explain the selection rules of [atomic spectroscopy](#).<sup>[17]</sup> The first character tables were compiled by László Tisza (1933), in connection to vibrational spectra. Robert Mulliken was the first to publish character tables in English (1933), and E. Bright Wilson used them in 1934 to predict the symmetry of vibrational [normal modes](#).<sup>[18]</sup> The complete set of 32 crystallographic point groups was published in 1936 by Rosenthal and Murphy.<sup>[19]</sup>

## Molecular nonrigidity

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As discussed above in the section **Point groups and permutation-inversion groups**, point groups are useful for classifying the vibronic states of *rigid* molecules (sometimes called *semi-rigid* molecules) which undergo only small oscillations about a single equilibrium geometry. Longuet-Higgins has introduced a more general type of symmetry group suitable not only for classifying the rovibronic states of rigid molecules but also for classifying the states of *non-rigid* (or *fluxional*) molecules that tunnel between equivalent geometries (called *versions*)<sup>[20]</sup> and which can also allow for the distorting effects of molecular rotation.<sup>[11]</sup> These groups are known as *permutation-inversion* groups, because the symmetry operations in them are energetically feasible permutations of identical nuclei, or inversion with respect to the center of mass (the [parity](#) operation), or a combination of the two.

For example, [ethane](#) (C<sub>2</sub>H<sub>6</sub>) has three equivalent [staggered](#) conformations. Tunneling between the conformations occurs at ordinary temperatures by [internal rotation](#) of one methyl group relative to the other. This is not a rotation of the entire molecule about the C<sub>3</sub> axis. Although each conformation has D<sub>3d</sub> symmetry, as in the table above, description of the internal rotation and associated quantum states and energy levels requires the more complete permutation-inversion group G<sub>36</sub>.

Similarly, [ammonia](#) (NH<sub>3</sub>) has two equivalent pyramidal (C<sub>3v</sub>) conformations which are interconverted by the process known as [nitrogen inversion](#). This is not the point group inversion operation *i* used for centrosymmetric rigid molecules (i.e., the inversion of vibrational displacements and electronic coordinates in the nuclear center of mass) since NH<sub>3</sub> has no inversion center and is not centrosymmetric. Rather it is the inversion of the nuclear and electronic coordinates in the molecular center of mass (sometimes called the parity operation), which happens to be energetically feasible for this molecule. The appropriate permutation-inversion group to be used in this situation is D<sub>3h</sub>(M) which is isomorphic with the point group D<sub>3h</sub>.

Additionally, as examples, the [methane](#) (CH<sub>4</sub>) and H<sub>3</sub><sup>+</sup> molecules have highly symmetric equilibrium structures with T<sub>d</sub> and D<sub>3h</sub> point group symmetries respectively; they lack permanent electric dipole moments but they do have very weak pure rotation spectra because of rotational centrifugal distortion.<sup>[21][22]</sup> The permutation-inversion groups required for the complete study of CH<sub>4</sub> and H<sub>3</sub><sup>+</sup> are T<sub>d</sub>(M) and D<sub>3h</sub>(M), respectively.

A second and less general approach to the symmetry of nonrigid molecules is due to Altmann.<sup>[23][24]</sup> In this approach the symmetry groups are known as *Schrödinger supergroups* and consist of two types of operations (and their combinations): (1) the geometric symmetry operations (rotations, reflections, inversions) of rigid molecules, and (2) *isodynamic operations*, which take a nonrigid molecule into an energetically equivalent form by a physically reasonable process such as rotation about a single bond (as in ethane) or a molecular inversion (as in ammonia).<sup>[24]</sup>

## See also

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- Parity (physics) § Molecules
- Irreducible representation § Applications in theoretical physics and chemistry
- Woodward-Hoffmann rules § Correlation diagrams
- Hapticity § Hapticity and fluxionality
- Character table
- Crystallographic point group
- Point groups in three dimensions
- Symmetry of diatomic molecules
- Symmetry in quantum mechanics

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## External links

- Point group symmetry (<https://www.staff.ncl.ac.uk/j.p.goss/symmetry/>) @ Newcastle University
- Molecular symmetry (<http://www.ch.ic.ac.uk/local/symmetry/>) @ Imperial College London
- Molecular Symmetry Online (<https://telem.openu.ac.il/symmetry>) @ The Open University of Israel
- Molecular Point Group Symmetry Tables (<http://www.webqc.org/symmetry.php>)
- Symmetry @ Otterbein (<http://symmetry.otterbein.edu/>)
- An internet lecture course on molecular symmetry @ Bergische Universitaet ([http://www.ptc.uni-wuppertal.de/FMS\\_Overview.html](http://www.ptc.uni-wuppertal.de/FMS_Overview.html))
- Character tables for point groups for chemistry Link ([http://gernot-katzers-spice-pages.com/character\\_tables/index.html/](http://gernot-katzers-spice-pages.com/character_tables/index.html/))

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