* Modified Euler's Method

Instead of approximating $f(x,y)$ by $f(x_0,y_0)$ in Equation (1)

$$y_1 = y_0 + \int_{x_0}^{x_1} f(x,y)\,dx \quad \text{—— (1)}$$

We now approximate the integral given in equation (1) by means of trapezoidal rule to obtain

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1)\right] \text{— (2)}$$

We thus obtain the iteration formula

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1^{(n)})\right] \text{— (3)}$$

Where $y_1^{(n)}$ is the n^{th} approximation to y_1.

The iteration formula (3) can be started by choosing $y_1^{(0)}$ from Euler's formula.

$$y_1^{(0)} = y_0 + h f(x_0,y_0).$$

Then $\quad y_n^{(0)} = y_{(n-1)} + h f(x_{n-1}, y_{n-1})$

Exp. Determine the value of y when $x=0.1$ given that $y(0)=1$ and $y' = \frac{dy}{dx} = x^2+y$

Solution. We take $h=0.05$, with $x_0=0$ and $y_0=1.0$ then we have

$$f(x_0,y_0) = x_0^2 + y = (0)^2+1 = 1.0$$

Hence Euler's formula gives

$$y_1^{(0)} = y_0 + h f(x_0,y_0)$$

$$y_1^{(0)} = 1.0 + 0.05(1) = 1.05$$

Further, $x_1 = x_0 + h = 0.0 + 0.05 = 0.05$ and

$$f(x_1, y_1^{(0)}) = (x_1)^3 + y_1^{(0)} = (0.05)^3 + 1.05 = 1.0525$$

The average of $f(x_0, y_0)$ and $f(x_1, y_1^{(0)})$ is 1.0262. The value of $y_1^{(1)}$ can therefore be computed by using in formula.

$$y_1^{(n+1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(n)})] \quad \text{—— (1)}$$

and we obtain

put $n=0$ in above formula. (1)

$$y_1^{(1)} = 1.0 + \frac{0.05}{2}[1.0 + 1.0525]$$

$$= 1.0 + \frac{0.05}{2}[2.0525]$$

$$= 1.0 + 0.05[1.02625] = 1.0 + 0.0513125$$

$$y_1^{(1)} = 1.0513$$

Repeating the procedure, we obtain $y_1^{(2)}$ from formula (1) as $y_1^{(2)} = 1.0513$. Hence we take $y_1 = 1.0513$ which is correct to four decimal places.

Next with $x_1 = 0.05$, $y_1 = 1.0513$ and $h = 0.05$ we continue the procedure to obtain y_2 i.e the value of y when $x = 0.1$. The results are

$$y_2^{(0)} = 1.1040, \quad y_2^{(1)} = 1.1055, \quad y_2^{(2)} = 1.1055$$

Hence we conclude that the value of y when $x = 0.1$ is 1.1055