* Method of variation of parameters to find P.I.

Consider the differential Equation

$$\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2 y = X \quad \text{---} \quad (1)$$

Let its C.F be $y = c_1y_1 + c_2y_2$

So that y_1 and y_2 statisfy the Equation

$$\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2 y = 0 \quad \text{---} \quad (2)$$

Assume that P.I of Eq. (1) is

$$y = uy_1 + vy_2 \quad \text{---} \quad (3)$$

where u and v are unknown functions of x.

Differentiating Eq (3) w.r.t. x, we get

$$y' = uy_1' + vy_2' + u'y_1 + v'y_2$$

on assuming that $u'y_1 + v'y_2 = 0$ ——— (4)

Then $y' = uy_1' + vy_2'$ ——— (5)

Again differentiating Eq (5) then we get

$$y'' = uy_1'' + vy_2'' + u'y_1' + v'y_2' \quad \text{---} \quad (6)$$

on substituting y'' and y from Eq (6) and 3 in Eq(1).

Then noting that y_1 and y_2 satisfy equation (2)

we obtain $u'y_1' + v'y_2' = X$ — (7)

Solving Eq (4) and (7), we get u', & v'.

Integrating u' & v', we have u & v. Thus the P.I of (3) is known.

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Exp. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$

Sol. Given diff. Eqn. in symbolic form is

$$(D^2+4)y = \tan 2x$$

→ To find C.F from $(D^2+4)y = 0$

Its Auxiliary Eqn is $D^2+4 = 0$

$$D = \pm 2i$$

$$C.F = c_1 \cos 2x + c_2 \sin 2x$$

→ To find P.I

Assume that P.I is

$$y = u\cos 2x + v\sin 2x \quad \text{—— (1)}$$

on differentiating Eqn (1)

$$y' = -2u\sin 2x + 2v\cos 2x + u'\cos 2x + v'\sin 2x \quad \text{—— (2)}$$

on taking $u'\cos 2x + v'\sin 2x = 0$ —— (3)

then Eqn (2) gives

$$y' = -2u\sin 2x + 2v\cos 2x \rightarrow (4)$$

Again differentiating Eq (4) then

$$y'' = -4u\cos 2x - 4v\sin 2x - 2u'\sin 2x + 2v'\cos 2x$$

Substituting the value of y'' and y given differential Eqn, we get

$$-4u\cancel{\cos 2x} - 4v\cancel{\sin 2x} - 2u'\sin 2x + 2v'\cos 2x + 4(u\cancel{\cos 2x} + v\cancel{\sin 2x}) = \tan 2x$$

$$-2u'\sin 2x + 2v'\cos 2x = \tan 2x \quad \text{—— (5)}$$

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Solving Eqns (3) and (5) for u' and v', we obtain

$$u' = -\frac{\sin^2 2x}{2\cos 2x} = -\frac{1}{2}\left[\frac{1-\cos^2 2x}{\cos 2x}\right]$$

$$v' = \frac{\sin 2x}{2} = \frac{1}{2}\cdot \sin 2x$$

$$u = -\frac{1}{2}\left[\int \sec 2x\,dx - \int \cos 2x\,dx\right]$$

$$u = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right]$$

$$u = \frac{1}{4}\left[\sin 2x - \log(\sec 2x + \tan 2x)\right]$$

$$v = \frac{1}{2}\int \sin 2x\,dx = -\frac{1}{4}\cos 2x$$

Putting u and v in Eqn (1) we get

$$P.I = \frac{1}{4}\left[\sin 2x - \log(\sec 2x + \tan 2x)\right].\cos 2x$$

$$- \frac{1}{4}\sin 2x.\cos 2x$$

$$P.I = -\frac{1}{4}\cos 2x.\log(\sec 2x + \tan 2x)$$

Hence, the general solution is

$$y = C.F + P.F$$

$$y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x)$$