Heat

(Maxwell velocity Distribution Law)

Lecture – 1

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Maxwell’s distribution law : The gas molecules moves with different velocity in different direction due to collisions The RMS velocity remains the same at a fixed temperature .

Some molecules are moving with a velocity higher than RMS velocity but others are lower than RMS velocity .

But the mean kinetic energy of all the molecules remains constant at a given temperature .Why this so ?

This interesting question was first answered by Maxwell in 1859 . These velocities are governed by a certain laws known as Maxwell distribution law .

Let n be the number of molecules per unit volume , if molecule move with a velocity c in an arbitrary direction , whose components are u,v and w along X , Y and Z direction respectively . Such that

$$c^2 = u^2 + v^2 + w^2 \qquad (1)$$

The number of molecules per unit volume having the velocities lying between u and u + du is given by

$$= n_u \, du$$

When n_u is some function of u i.e. ,

$$n_u = n\, f(u)du$$

and hence the probability that a molecule selected at random will have the velocities lying between u and u+du is

$$= f(u)du$$

We assume that f(u) is independent of v and w .Similarly , the probability for molecules with velocities lying between v and v + dv is

$$= f(v)dv$$

and that with velocities lying between w and w + dw is

$$= f(w)dw$$

Hence the total probability that a molecule have the velocity components lying between u and u + du , v and v + dv and w and w + dw is

$$= f(u)f(v)f(w)du\ dv\ dw , \qquad (2)$$

Which is the product of the individual probabilities .

Then the number of such molecules per unit volume will be

$$= nf(u)f(v)f(w)dudvdw \quad (3)$$

If c be the resultant velocity , so the number of these molecules must be

$$= n\,F(c)\,du\,dv\,dw \quad (4)$$

Where F (c) is the probability for molecules with velocity c

Equating Eq.(3) and (4) ,we get

$$f(u)f(v)f(w) = F(c) = \phi(c^2) \quad (5)$$

where $\phi(c^2)$ is a new function of c^2 , For a fixed value of c , $\phi(c^2)$ is constant , hence

$$d[f(u)f(v)f(w)] = d[\phi(c^2)] = 0$$

or

$$f'(u)du\, f(v)\, f(w) + f(u)f'(v)\, dv\, f(w) + f(u)\, f(v)f'\, dw = 0 \qquad (6)$$

where $f' = \frac{df}{du}$,etc dividing by f(u)f(v)f(w) , we get

$$\frac{f'(u)}{f(u)}\, du + \frac{f'v}{f(v)}\, dv + \frac{f'w}{f(w)}\, dw = 0 \qquad (7)$$

For fixed value of c , we have Eq. (1)

udu + vdv + wdw = 0 (8)

Multiplying Eq. (8) by β and adding to Eq. (7) , which is known as Laplace's method of undetermined multiplier ,we get

$$[(\frac{f\prime(u)}{f(u)} + \beta u) du + (\frac{f'v}{f(v)} + \beta v) dv + (\frac{f\prime w}{f(w)} + \beta w) dw] = 0 \quad (9)$$

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$$\frac{f\prime(u)}{f(u)} = - \beta u \qquad (10a)$$

$$\frac{f'v}{f(v)} = - \beta v \qquad (10b)$$

$$\frac{f\prime w}{f(w)} = - \beta w \qquad (10c\ )$$

From Eq. (10a)

$$\frac{df(u0}{f(u)} = \beta u\ du$$

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$$\frac{df(u_0)}{f(u)} = \beta u \, du$$

On integration ,we have

Ln f(u) - ½ βu^2 + ln a

or ,

$$f(u) = ae^{-\beta u2\,/2} = a\,e^{-bu2} \quad , \qquad (11)$$

a and b are constants . similarly Eq.(10b) and Eq.(10c) give

$$f(v) = a\, e^{-bv2}\ , \qquad (12)$$

$$f(w) = a\, e^{-bw2}\ , \qquad (13)$$

Hence the number of molecules having the velocity components lying between u and u + du , v and v + dv and w and w + dw is given by

$$dn = na^{3}\, e^{-b(u2+v2+w2)}\ dudvdw\ , \qquad (14)$$

where a,b and c are constants . This is called Maxwell's law for velocity components . Here dudvdw is volume element in the velocity space .