Maxwell thermodynamic relation

This relation exist between different thermodynamic functions U, H , A and G are expressed as functions of suitably two independent variables P, V, T and S .

First thermodynamic relation

$$(dT/dV)_S = -(dP/dS)_V$$

Proof : In terms of internal energy , the law of thermodynamics applied as

$TdS = dU + PdV$

$dU = TdS - PdV$ (1)

Where U is an internal energy , S and V are independent variables

Since dU is an exact differential , it follows from (1)

$T = (dU/dS)_V$

$P = (dU/dV)_S$

and

$(dT/dV)_S = - (dP/dS)_V$

This is called Maxwell First Thermodynamic relation .

Second Thermodynamic relation

$(dT/dP)_S = (dV/ dS)_P$

Proof :In terms of enthalpy H or Total Heat

The enthalpy is defined as

$$dH = dU + d(PV)$$

$$= dU + PdV + VdP$$

$$= TdS + VdP \qquad (2)$$

Since

$TdS = dU + PdV$

Here H is a function of independent variables S and P. since dH is an exact differential , from (2)

We obtain

$T = (dH/dS)_P$

$V = (dH/dP)_S$ (3)

and

$(dT/dP)_S = (dV/dS)_P$

This is called Maxwell Second Thermodynamic relation .