

Thermal Physics
(Maxwell Distribution Law)
Lecture - 6

Dr. Tarun Kumar Dey,
Associate professor

Department of Physics ,

L.S College; BRA Bihar University, Muzaffarpur

Youtube channel – Tarun Kumar Dey

Online Course Link :

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In spherical polar coordinate (c, θ, ϕ) the volume element can be written as

$$dudvdw = c^2 dc \sin\theta d\theta d\phi .$$

substituting this value in (14) , we get the number of molecules having velocities lying between c and $c + dc$, $\theta + d\theta$ and $\phi + d\phi$

$$dn = na^3 e^{-bc^2} c^2 dc \sin\theta d\theta d\phi . \quad (15)$$

Integrating this equation over θ and ϕ with the limits $\theta = 0$ to $\theta = \pi$ and $\phi = 0$ to $\phi = 2\pi$ we get an expression for number of molecules having the velocities lying between c and $c + dc$. Thus

$$\begin{aligned} dn_c &= na^3 e^{-bc^2} c^2 dc \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \\ &= 4\pi a^3 e^{-bc^2} c^2 dc \end{aligned} \tag{16}$$

This is known as Maxwell distribution law .

It gives the complete knowledge of the gas only in the statistical sense

from this probability of its velocity lying between c and $c + dc$ is

$$f(c) dc = 4\pi a^3 e^{-bc^2} c^2 dc \quad (17)$$

Let $(bc^2)^{1/2} = X$ then (17) becomes

$$f(X) dX = 4\pi^{-1/2} e^{-X} X^2 dX$$

Or ,

$$f(X) = 4\pi^{-1/2} e^{-X} X^2 . \quad (18)$$

This equation is universal . In this form , the distribution function depends neither on the kind of gas nor on the temperature .

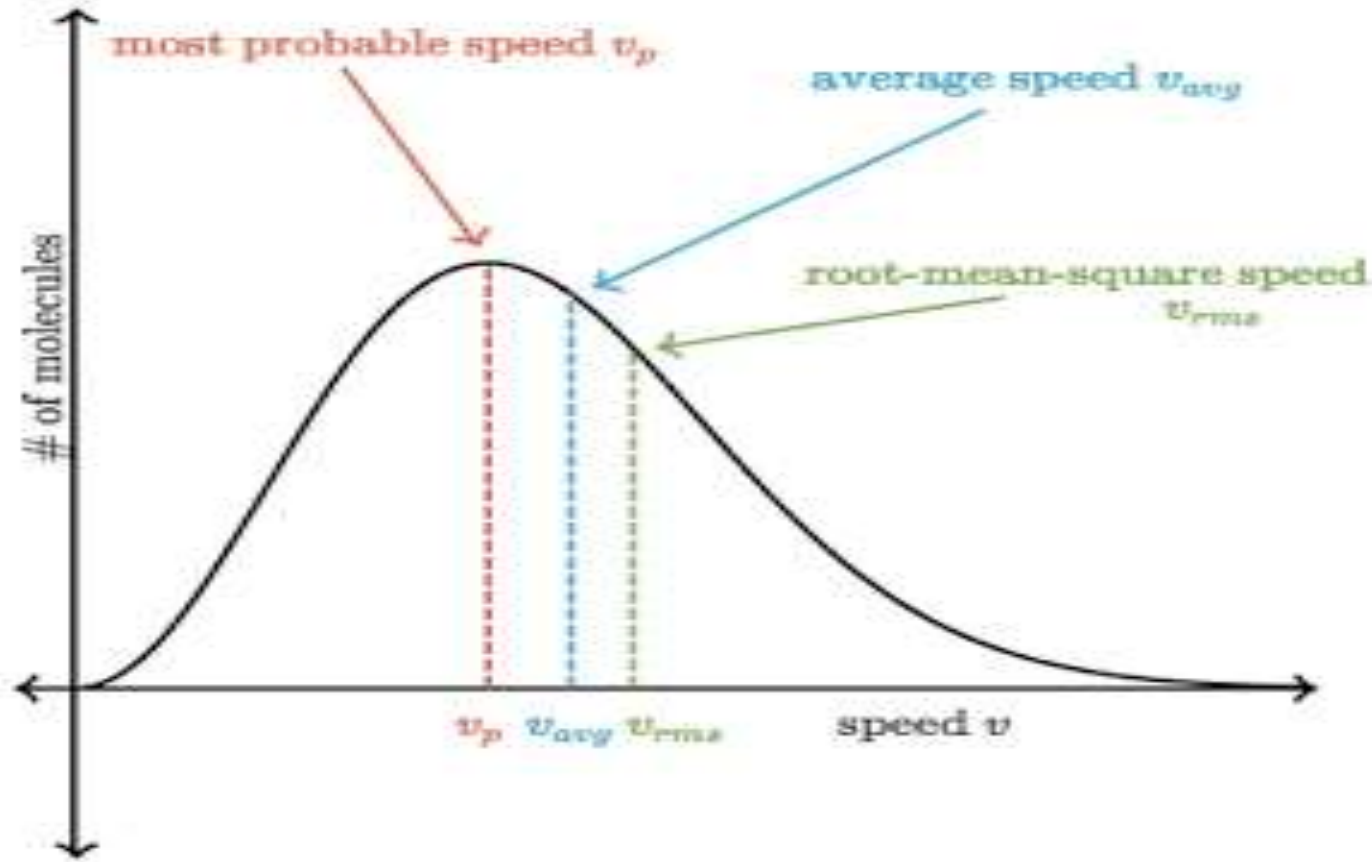


Figure 1 . The Maxwell distribution function as a function of speed .

From fig 1 , we obtain the probability when $X = 1$ is the greatest .Thus the gas has the most probable velocity $X = 1$

Determination of the constants :

Since total number of molecules per unit volume having velocities 0 and ∞ is in n , we have

$$4 \pi n a^3 \int_0^{\infty} e^{-bc^2} c^2 dc = n \quad (19)$$

Integrating this we can determine a

Let us put

$$bc^2 = X$$

$$dc = \frac{1}{2} b^{-1/2} X^{-1/2} dX ,$$

Eq(19) becomes

$$2\pi na^3 \frac{1}{2} b^{-1/2} \int_0^\infty e^{-X} X^{1/2} dX = n$$

Or ,

$$a = \left(\frac{b}{\pi} \right)^{1/2} \tag{20}$$