Matrices

(iv) show that matrix product is associative

Let A, B, C be matrices such that (AB) and $(AB)c$ are both defined. Then the required result will be proved if we show that

(a) $A(Bc)$ is defined; a and

(b) $A(Bc) = (AB)c$

Proof :- Let $A = (a_{ij})_{m,n}$ and $B = (b_{jk})_{n,p}$

Then AB is defined and we have

$$AB = \left(\sum_{j=1}^{n} a_{ij} b_{jk}\right)_{m,p}$$

To make $(AB)c$ meaningful we take $c = (c_{kl})_{p,q}$ so that

$$(AB)c = \left(\sum_{k=1}^{p}\left\{\left(\sum_{j=1}^{n} a_{ij} b_{jk}\right) c_{kl}\right\}\right)_{m,q} \quad \text{--- (1)}$$

Now, from the forms of B and C we see that BC is defined and

$$BC = \left(\sum_{k=1}^{p} b_{jk} c_{kl}\right)_{n,q} \quad \text{--- (2)}$$

Also, from the forms of A and BC, we see that $A(Bc)$ is defined. This proves (a).

Finally, we have $A(BC) = \left(\sum_{j=1}^{n} a_{ij} \left\{ \sum_{k=1}^{p} b_{jk} c_{kl} \right\} \right)_{m,q}$

$$= \left(\sum_{j=1}^{n} \sum_{k=1}^{p} a_{ij} (b_{jk} c_{kl}) \right)_{m,q}$$

$$= \left(\sum_{j=1}^{n} \sum_{k=1}^{p} (a_{ij} b_{jk}) c_{kl} \right)_{m,q}$$

[$\because$ product of scalar is associative]

$$= \left(\sum_{k=1}^{p} \left\{ \left(\sum_{j=1}^{n} a_{ij} b_{jk} \right) c_{kl} \right\} \right)_{m,q}$$

$= (AB)\,C$ (by (i))

This proves (b).