## Cramer's Rule :-

Cramer has given a formula for solving the non homogenious equation $AX = B$ if $A$ is a non - singular (square) matrix [i.e. if the number of equations = the number of the unknowns.]

We find the formula when $A$ is a $3\times3$ non-singular matrix

Let the system of equations be

$$a_1 x + b_1 y + c_1 z = d_1 \quad \text{—— (1)}$$
$$a_2 x + b_2 y + c_2 z = d_2 \quad \text{—— (2)}$$
$$a_3 x + b_3 y + c_3 z = d_3 \quad \text{—— (3)}$$

Then $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

Let $A_1, B_1, C_1, \ldots$ be respectively the co-factors of $a_1, b_1, c_1 \ldots$ in $\Delta$, then we know that

$$a_1 A_1 + a_2 A_2 + a_3 A_3 = \Delta,$$
$$b_1 B_1 + b_2 B_2 + b_3 B_3 = \Delta \text{ etc.}$$
and $$b_1 A_1 + b_2 A_2 + b_3 A_3 = 0,$$
$$b_1 C_2 + b_3 C_2 + b_3 C_3 = 0 \text{ etc.}$$

Now multiplying (1) by $A_1$, (2) by $A_2$, (3) by $A_3$ and adding we get

$$(a_1A_1 + a_2A_2 + a_3A_3)x + (b_1A + b_2A_2 + b_3A_3)y + (c_1A_1 + c_2A_2 + c_3A_3)z$$

$$= d_1A_1 + d_2A_2 + d_3A_3$$

or, $\Delta \cdot x + 0 \cdot y + 0 \cdot z = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

or, $x = \dfrac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\Delta}$

lly multiplying (1), (2), (3) by $B_1$, $B_2$, $B_3$ respectively and adding we get

$$y = \dfrac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

and multiplying (1), (2), (3) by $C_1$, $C_2$, $C_3$ respectively and adding we get

$$z = \dfrac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

## Solution by matrix-method :-

To solve (I) we put it in the matrix form $AX = B$ (I') is stated above.

**Method (i)** :- we reduce A to its echelon form by performing elementary row operations on both sides of (I'), remembering that every elementary row operation on B (i.e. on the product AX) is equivalent to the same elementary row operation on A (i.e. on the pre factor of product AX).

**Method (ii)** :- This method is applicable only when A is a non-singular matrix.

In this case $A^{-1}$ exists.

Pre multiplying (I') be $A^{-1}$ we get

$$A^{-1}(AX) = A^{-1}B$$

$$\text{or,} \quad X = A^{-1}B$$

This is the required solution.

**Example (i)**

show that the equations

$$x - 3y + 2z + 4 = 0$$
$$2x + y + 4z + 1 = 0$$
$$3x + 2y + 5z - 1 = 0$$
$$2y + z = 0$$
$$3x - 2y + 6z + 5 = 0$$

are consistent and solve them.