Then (I) can be written in the matrix form $AX = B$ (I')

Now $|A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= 2(1+1) + 1(1-1) + 3(-1-1)$

$= -2 \neq 0$

$\therefore A^{-1}$ exists.

Let A_1, A_2 etc. be co-factors of a_1, a_2 etc. in $|A|$.

Then, $A_1 = +\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$, $A_2 = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$A_3 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$

$B_1 = -\begin{vmatrix} -1 & 3 \\ -1 & 1 \end{vmatrix} = -2$, $B_2 = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$, $B_3 = -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}$
$= 1$

$C_1 = \begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = -4$, $C_2 = -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1$

$C_3 = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$

$$\therefore A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 2 & -2 & -4 \\ 6 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

Now from (I') we have $X = A^{-1}B$

i.e. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -9 & +6 & +4 \\ 0 & +\frac{6}{2} & -3 \\ 9 & -3 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\therefore x = 1$, $y = 2$, $z = 3$

This is the required solution.

[Note :- In order to show that the equation are Consistent, we proceed as follows:-

We have $|A| = -2 \neq 0$

$\therefore$ rank $A = 3$

Also $[AB] = \begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{bmatrix}$ This is a 3×4 matrix. The highest available order of a minor of $[AB]$ is 3. Now a third order minor of $[AB]$ is $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \neq 0$ $\therefore$ rank $[AB] = 3$ $=$ rank A

$\therefore$ The equations are Consistant.]

Matrix method (By elementary row operations):-

Here the co-efficient matrix is $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

The matrix of constants is $B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$

$\therefore$ Augmented matrix is

$$[A : B] = \left[\begin{array}{ccc:c} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$$R_1 \to R_1 - R_2 \left[\begin{array}{ccc:c} 1 & -2 & 2 & 3 \\ 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & 4 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to -\frac{1}{2} R_3 \end{array} \left[\begin{array}{ccc:c} 1 & -2 & 2 & 3 \\ 0 & +3 & -1 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right]$$

$$\begin{array}{l} R_1 \to R_1 + 2R_3 \\ R_2 \to R_2 - 2R_3 \end{array} \left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 2 \end{array}\right] R_3 \to R_3 - R_2 \left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$R_1 \to R_1 - 2R_3 \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Hence rank $[AB]$ = rank $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3 =$ rank A

$\therefore$ The given equation are Constant.