Ans. The given system of equations is equivalent to

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & 2 & 1 \\ 3 & -2 & 6 \end{bmatrix} X = \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \\ -5 \end{bmatrix} \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

we successively apply row transformation on the pre factor on L.H.S. and the product matrix on R.H.S.

By $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_5 \to R_5 - 3R_1$ we get

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 7 & 0 \\ 0 & 11 & -1 \\ 0 & 2 & 1 \\ 0 & 7 & 0 \end{bmatrix} X = \begin{bmatrix} -4 \\ 7 \\ 13 \\ 0 \\ 7 \end{bmatrix}$$

By $R_3 \to R_3 - \frac{11}{7}R_2$, $R_4 \to R_4 - \frac{2}{7}R_2$, $R_5 \to R_5 - R_2$ we get

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 7 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} -4 \\ 7 \\ 2 \\ -2 \\ 8 \end{bmatrix}$$

By $R_4 \to R_4 + R_3$ we get

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 7 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} -4 \\ 7 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

By $R_1 \to R_1 + 2R_3$ and $R_2 \to \frac{1}{7} R_2$ we get

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

It follows that rank of the Co-efficient matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & 2 & 1 \\ 3 & -2 & 6 \end{bmatrix}$

$$= \text{rank} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 3$$

and if B is the matrix of Constants,

i.e. if $B = \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \\ 5 \end{bmatrix}$, then $\text{rank}[A \vdots B] = \text{rank} \begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$= 3$