Matrices

(v) show that matrix product is distributive over matrix addition.

The theorem will be proved if we show that whenever $B+C$ and $A(B+C)$ are defined then.

(a) AB, AC and $AB+AC$ are defined; and

(b) $A(B+C) = AB+AC$

where A, B, C are matrices.

Proof:— Let $B = (b_{jk})_{n,p}$ and

$C = (c_{jk})_{n,p}$ be matrices

so that $B+C$ is defined and $B+C = (b_{jk}+c_{jk})_{n,p}$

Let $A = (a_{ij})_{m,n}$ be a matrix so that

$A(B+C)$ is defined

and $$A(B+C) = \left(\sum_{j=1}^{n}\left\{a_{ij}\,(b_{jk}+c_{jk})\right\}\right)_{m,p} \quad \text{— (1)}$$

Now, from the forms of A, B and A, C we see that

$$AB \text{ is defined and } AB = \left(\sum_{j=1}^{n} a_{ij}\, b_{jk}\right)_{m,p} \quad \text{— (2)}$$

and also AC is defined and $$AC = \left(\sum_{j=1}^{n} a_{ij}\, c_{jk}\right)_{m,p} \quad \text{— (3)}$$

Moreover, since AB and AC are matrices of the same type

$\therefore AB + AC$ is also defined.

Thus we see that whenever $B+C$ and $A(B+C)$ are defined.

Then AB, AC and $AB+AC$ are also defined.

This proves (a).

Finally, we have $AB + AC = \left(\sum_{j=1}^{n}\{a_{ij}b_{jk} + a_{ij}c_{jk}\}\right)_{m,p}$

[by (2) & (3)]

$= \left\{\sum_{j=1}^{n} a_{ij}(b_{jk} + c_{jk})\right)_{m,p}$

[$\because$ scalar multiplication is distributive over scalar addition]

$= A(B+C)$, by (1)

This proves (b).