LINEAR PROGRAMMING PROBLEMS AND THEIR GRAPHICAL SOLUTIONS :-

Linear programming deals with that class of programming problems for which all relations among the variables are linear.

The relations must be linear both in the constraints and in the function to be optimized. The term linear means that all the relations in the particular problem are linear and the term programming' refers to the process determining a particular programme or plan of action.

General linear programming problems :-

A linear programming problem includes a set of simultaneous linear equation which represent the conditions of the problem and a linear function which express the objective function of the problem.

The linear function which is to be optimized is called the objective function and the conditions of the problem expressed as simultaneous linear equations (or inequalities) are referred as constraints.

A general linear programming problem can be stated as follows:

Find $x_1, x_2, \ldots, x_n$ which optimize the linear function

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \quad \text{--- (1)}$$

subject to the Constraints

$$
\left.
\begin{array}{l}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1j}x_j + \cdots + a_{1n}x_n \ (\leq = \geq)\ b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2j}x_j + \cdots + a_{2n}x_n \ (\leq = \geq)\ b_2 \\
\text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \\
\text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \\
a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{ij}x_j + \cdots + a_{in}x_n \ (\leq = \geq)\ b_i \\
\text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \\
\text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \quad \text{--} \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mj}x_j + \cdots + a_{mn}x_n \ (\leq = \geq)\ b_m
\end{array}
\right] - (2)
$$

and non-negative restrictions

$$x_j \geq 0, \quad j = 1, 2, \cdots, n \quad \text{———} \quad (3)$$

where all a_{ij}'s b_i's and c_j's are Constants and x_j's are variables.

In the Conditions given by (2) there may be any of the three signs $\leq, =, \geq$. The function Z given by (1) is Called the objective function and the Conditions given by (2) are termed as the Constraints of the linear programming problem.

We shall always assume that all $b_i \leq 0$. If any one is negative, we make it positive by multiplying both sides of the Corresponding inequality by -1. By this multiplication the inequality is also reversed.

The above linear programming problem may also be stated in matrix as follows:

optimize $Z = cx$

subject to

$$Ax \, (\leq, =, \geq) \, b$$

and $x \geq 0$

where $A = [a_{ij}]$ is the matrix of Coefficients of order $m \times n$

$c = (c_1, c_2, \ldots, c_n)$ is a row vector Known as price vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} = [x_1, x_2, \ldots, x_n]^*$$

is Column vector of variables.

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_m \end{bmatrix} = [b_1, b_2, \ldots, b_m]$$

is Column vector Called the requirement vector, and 0 is n dimensional null Column vector.

The Column vector formed by the Coefficients of x_j in all the Constraints is denoted by α_j

i.e. $$\alpha_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \cdots \\ a_{mj} \end{bmatrix} = [a_{1j}, a_{2j}, \ldots, a_{mj}]$$

then $$A = (\alpha_1, \alpha_2, \ldots, \alpha_n)$$

<u>Note</u>:- In a general linear programming problem it is assumed that the number of rows of Coefficient matrix A is less than its number of Columns.