*Continue of previous Theorem
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Conversely, (for, T is isomorphism of U onto V)

Let U(F) and V(F) be two isomorphism finite dimensional vector spaces.

Then to prove that dim U ≅ dim V

Let dim U = n, let $S = \{u_1, u_2, ---- u_n\}$ be basis of U

if T is an isomorphism of U onto V, we shall show that $S^* = \{T(u_1), T(u_2) \cdots T(u_n)\}$ is basis of V.
or $S^* = \{v_1, v_2, \ldots v_n\}$

Then V will also be of dimension n.

Firstly, we prove that S^* is L.I. (linear Ind.

then

$$\alpha_1 T(u_1) + \alpha_2 T(u_2) + \cdots \alpha_n T(u_n) = 0'$$

(zero vector of V)

$\alpha_i \in F \quad (i = 1, 2 \cdots n)$

$$\Rightarrow T(\alpha_1 u_1 + \alpha_2 u_2 + \cdots \alpha_n u_n) = 0'$$

∵ T is L.T.

$$\Rightarrow \alpha_1 u_1 + \alpha_2 u_2 + \cdots \alpha_n u_n = 0$$

∵ T is one-one $T(0) = 0$ where 0 is zero vector of U.

$\alpha_1 = 0, \alpha_2 = 0 \cdots \alpha_n = 0$

Since $u_1, u_2, \cdots u_n$ are L.I.

∴ S^* is Linearly independent.

Now to prove that $V = L(S^*)$, for this we prove that any vector $v \in V$ can be expressed as a linear combination of the vectors of set S^*.

Since, T is onto V, therefore $v \in V \Rightarrow$ there exists $u \in U$ such that

$$T(u) = v$$

Let $u = \alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 + \cdots \alpha_n u_n$

Then $v = T(u) = T(\alpha_1 u_1 + \alpha_2 u_2 + \cdots \alpha_n u_n)$

$$v = \alpha_1 T(u_1) + \alpha_2 T(u_2) + \cdots \alpha_n T(u_n)$$

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots \alpha_n v_n$$

Thus v is a linear combination of the vectors of $S^* = \{v_1, v_3 - \cdots v_n\}$

or $S^* = \{T(u_1), T(u_2), \cdots T(u_n)\}$

Hence $V = L(S^*)$. Therefore S^* is a basis of V. ~~Since S^* denote~~

Since S^* has dimension n vectors therefore $\dim V = n$