Linear Transformation

⇒ Linear Transformation (L.T.) is a function from one vector space V to another vector space W.

* Functions are mapping and transformation between two vector spaces, a domain V and a range W, that assigns to each element in V exactly one element in W.

i.e. functions that map a vector space V into a vector space W, and denoted by as

$$T : V \to W$$

Exp. V is called the domain of T

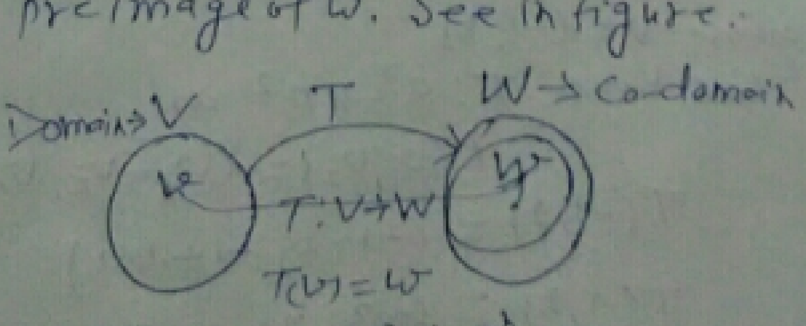
W is called the co-domain of T

If v is in V and w is in W such that

$T(v) = w$

then w is called the image of v under T.

The set of all images of vectors in V is called the range of T, and the set of all v in V such that $T(v) = w$ is called the preimage of w. See in figure.



$v = \{v_1, v_2, \cdots v_n\}$ in R^n

$T(v) = T(v_1, v_2, \cdots v_n)$

Def. of Linear Transformation - Let V and W be vector spaces. The function $T: V \to W$ is called linear transformation of V into W if the following two conditions/properties are true/hold as.

(i) $T(u+v) = T(u) + T(v)$, $\forall\ u, v \in V$

(ii) $T(\alpha v) = \alpha T(v)$ $\quad v \in V$ and $\alpha \in F$ is a scalar.

In other word, the conditions (i) & (ii) can be combined into a single condition:

$$T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$$

where $\alpha, \beta \in F$ & $u, v \in V$

Linear transformation (T) is also known as homomorphism of V into W.

If T is a homomorphsim of V onto W then W is called the homomorphic image of V.

Exp for any vector $v = (v_1, v_2)$ in R^2

let $T: R^2 \to R^2$ be defined by

$$T(v_1, v_2) = (v_1 - v_2,\ v_1 + 2v_2)$$

(a) find the image of $v = (-1, 2)$

(b) find the preimage of $w = (-1, 11)$

Sol. (a) for $v = (-1, 2)$

we have $T: R^2 \to R^2$ as

$$T(v_1, v_2) = (v_1 - v_2,\ v_1 + 2v_2)$$

$$T(-1, 2) = (-1-2,\ -1+2\times 2)$$

$T(-1, 2) = (-3, 3)$ is the image of $(-1, 2)$

(b) If $T(v) = (v_1 - v_2, v_1 + 2v_2) = (-1, 11)$

Then $v_1 - v_2 = -1$ —(i)

$v_1 + 2v_2 = 11$ —(ii)

from (ii) − (i)

$3v_2 = 12$

$v_2 = 4$

and $v_1 = 3$

$w = (3, 4)$

$w = (-1, 11)$

So the preimage of $w = (-1, 11)$ is $v = (3, 4)$ in R^2

* Hence functions (from one vector space to another) that preserve the operations of vectors addition and scalar multiplication, are called linear transformations.