* Linear Differential Equations of second-order with constant coefficients:-

(1) Homogeneous Equations:- A second-order linear differential equation with constant coefficients p and q without the right hand side (i.e $=0$) is of the form:-

$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$$

or $y'' + py' + qy = 0$ ——— (i)

If k_1 and k_2 are roots of the characterstic equation $\varphi(k) = k^2 + pk + q = 0$, then the general solution of Eqn (i) is written in one of the following:-

(i) if k_1 and k_2 are real and $k_1 \neq k_2$

$$\Rightarrow y = c_1 e^{k_1 x} + c_2 e^{k_2 x}$$

(ii) if k_1 and k_2 are real and equal $k_1 = k_2$

$$\Rightarrow y = e^{k_1 x}(c_1 + c_2 x)$$

(iii) if k_1 and k_2 are imaginary i.e

$k_1 = \alpha + \beta i$ and $k_2 = \alpha - \beta i$ $(\beta \neq 0)$

$$\Rightarrow y = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$$

Exp. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

Sol. Given Eqn compares with

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

$P = 2$ & $Q = 1$

∴ The auxiliary Equation (or characterstic

$\Rightarrow \quad m^2 + 2m + 1 = 0$

$(m+1)^2 = 0 \quad \Rightarrow (m+1)(m+1) = 0$

$m = -1$ and $m = -1$

$m_1 = m_2 = -1$ are real and equal

The general solution is

$$y = e^{-x}(c_1 + c_2 x)$$

Exp. Solve $y'' - 4y' + 5y = 0$

Solution The auxilary Eqn (or characterstic Eqn) of the given equation.

$m^2 - 4m + 5 = 0$

The roots from quadratic Eqn:- $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

~~m=b±~~ $a = 1, b = -4$ and $c = 5$

$$m = \frac{+4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2 \times 1} = \frac{+4 \pm 2i}{2} = (+2 \pm i)$$

$m_1 = 2 + i$ and $m_2 = 2 - i$

The general solution

$$y = e^{2x}[c_1 \cos x + c_2 \sin x]$$