13/04/2021

Q.1 **Exp.** Evaluate $\lim_{x\to 0} x \log x$

Sol. This is of the form $0\times\infty$ when $x\to 0$

$\because$ The given limit $= \lim_{x\to 0} \frac{\log x}{x^{-1}}$

using L. Hospital rule.

$$= \lim_{x\to 0} \frac{1/x}{-x^{-2}}$$

$$= \lim_{x\to 0} \frac{-1/x}{1/x^2}$$

$$= \lim_{x\to 0} \frac{-x\cancel{x}}{\cancel{x}} \Rightarrow \lim_{x\to 0} (-x) = 0$$

$\because \lim_{x\to 0} x\log x = 0$

Q.5. **Exp.** Find limit value of

$$\lim_{x\to 0} x^m (\log x)^n$$

Sol. This is of the form $0\times\infty$ when limit $x\to 0$.

$\because$ The given limit $\lim_{x\to 0} x^m (\log x)^n = \lim_{x\to 0} \frac{(\log x)^n}{x^{-m}} \left[\frac{\infty}{\infty}\right]$

By L. Hospital rule

$$\lim_{x\to 0} \frac{n(\log x)^{n-1}\cdot\frac{1}{x}}{-m x^{-m-1}} = \lim_{x\to 0} \frac{n(\log x)^{n-1}}{-m x^{-m}} \left[\frac{\infty}{\infty}\right]$$

Again L. H. rule.

$$= \lim_{x\to 0} \frac{n(n-1)(\log)^{n-2}\, 1/x}{-m\cdot(-m)x^{-m-1}} = \lim_{x\to 0} \frac{n(n-1)(\log x)^{n-2}}{(-m)^2\cdot x^{-m}}$$

$$\lim_{x \to 0} \frac{n(n-1)(\log x)^{n-2}}{(-m)^2 \cdot x^{-m}} \quad \left[\frac{\infty}{\infty}\right]$$

Again by L·Hospital rule applying till n times then numerator an denominator n times

$$= \lim_{x \to 0} \frac{n(n-1)(n-2)\cdots 1}{(-m)^n \cdot x^{-m}}$$

$$= \lim_{x \to 0} \frac{\lfloor n}{(-m)^n} \cdot x^m$$

$$= \frac{\lfloor n}{(-m)^n} \cdot (0)^m = 0$$

Hence.

$$\lim_{x \to 0} x^m \cdot (\log x)^n = 0$$