Exp. prove that

$$\lim_{x\to 0} \frac{xe^{x} - \log(1+x)}{x^2} = 3/2$$

Sol: Given that

$$\lim_{x\to 0} \frac{xe^{x} - \log(1+x)}{x^2}$$

Putting $x=0$, the given expression $\left[\frac{0}{0}\right]$

This is indeterminate form.

Now, the given expression

$$\lim_{x\to 0} \frac{xe^{x} - \log(1+x)}{x^2}$$

$$\lim_{x\to 0} \frac{x\left(1+x+\frac{x^2}{2}+\cdots\right) - \left(x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots\right)}{x^2}$$

Using the expansion of e^x and $\log(1+x)$

$$e^x = 1+x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots$$

$$\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$

Thus, $$\lim_{x\to 0} \frac{\left(x+x^2+\frac{x^3}{2}+\cdots\right) - \left(x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots\right)}{x^3}$$

$$\lim_{x\to 0} \frac{\left(x^2+\frac{x^2}{2}\right)+x^3\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{x^4}{3}+\frac{x^4}{4}\right)+\cdots}{x^2}$$

$$\lim_{x\to 0} \left(1+\tfrac{1}{2}\right)+x\left(\tfrac{1}{6}\right)+x^2\left(\tfrac{7}{12}\right)+\cdots$$

Applying limit $x\to 0$ then

$$= 3/2 + 0 + 0 \cdots$$

$$\lim_{x\to 0} \frac{xe^{x}-\log(1+x)}{x^2} = 3/2$$ Hence.

Exp. Evaluate $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2}\log(1+x)\right]$

Solution

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2}\log(1+x)\right]$$

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right)\right]$$

$$\lim_{x \to 0} \left[\cancel{\frac{1}{x}} - \cancel{\frac{1}{x}} + \frac{1}{2} - \frac{x}{3} + \frac{x^2}{4} - \cdots\right]$$

$$\lim_{x \to 0} \left(\frac{1}{2} - \frac{x}{3} + \frac{x^2}{4} - \frac{x^3}{5} + \cdots\right)$$

$$= \frac{1}{2} - 0 + 0 - \cdots$$

$$= \frac{1}{2}$$

Exp. Evaluate $\lim_{x \to \pi/2} \frac{x\sin x - \pi/2}{\cos x}$

Sol. Given expression

$$\lim_{x \to \pi/2} \frac{x\sin x - \pi/2}{\cos x}$$

Applying limit then

$$\frac{\pi/2 \cdot \sin \pi/2 - \pi/2}{\cos \pi/2} = \frac{0}{0}$$

This is indeterminate from $\left[\frac{0}{0}\right]$ then using L. Hospital rule.

$$\lim_{x \to \pi/2} \frac{\frac{d}{dx}(x\sin x - \pi/2)}{\frac{d}{dx}\cos x} = \lim_{x \to \pi/2} \frac{x.\cos x + \sin x}{-\sin x}$$

$$= \frac{\pi/2 \cdot \cos \pi/2 + \sin \pi/2}{-\sin \pi/2}$$

$$\lim_{x \to \pi/2} \frac{x\sin x - \pi/2}{\cos x} = -1$$