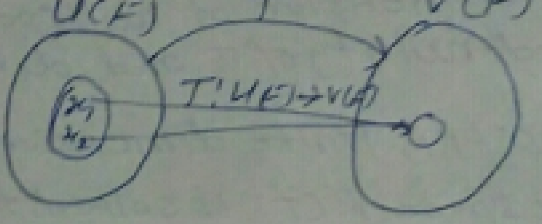
*① 13/04/21

Kernel of a Homomorphism (L.T): Let T be a homomorphism of a vector space U into a vector space V over field (F) (i.e $T: U \to V$). Then the set K of all the vectors/elements of U which are mapped into zero (0) element of V is called kernel of homomorphism and is defined as

$$K = \{x \in U : T(x) = 0, \text{ where } 0 \text{ is the zero vector of } V\}$$



and denoted by $\ker T$ or $N(T)$

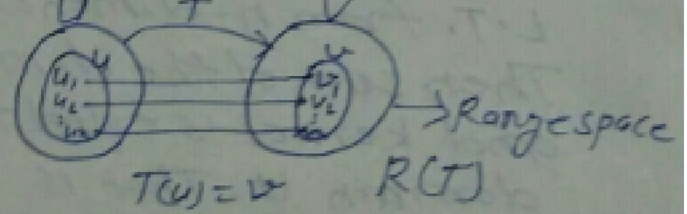
$\ker T$ is also called the null space of T

Def: Let $T: U \to V$ be L.T. Then the set of all vectors u in U that satisfy $T(u)=0$ is called $\ker T$.

* **Range and Null Space of a Linear Transformation**

Let U and V be two vector spaces and T be a linear transformation from U into V i.e $T: U \to V$. Then the image or range of T denoted as $R(T)$ is the set of all vectors $v \in V$ such that

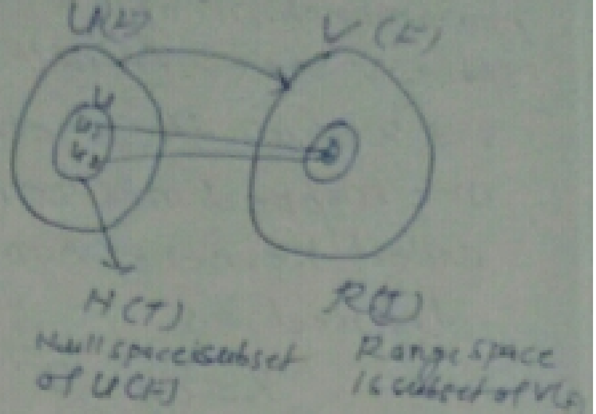
$$R(T) = \{v \in V : T(u) = v, \text{ for some } u \in U\}$$



and null space of T denoted as $N(T)$ is the set of all vector $u \in U$ such that

$T(u) = 0$ i.e image of u is zero.

$N(T) = \{ u \in U : T(u) = 0 \in V \}$



(i) $\ker(T) \subset U(F)$

(ii) $N(T) \subset U(F)$

(iii) $R(T) \subset V(F)$

→ If $U(F)$ is finite dimensional, then then dimension of nullspace $N(T)$ is called nullity of T and denoted $\eta(T)$

→ If $V(F)$ is finite dimensional, then the range space $R(T)$ is called the rank of T and denoted by $\rho(T)$.

Rank and Nullity of a Linear Transformation

Let $T: U(F) \rightarrow V(F)$ be a linear transformation where $U(F)$ is finite dimensional vector space. The dimension of the kernel of T is called nullity of T and denoted by $\eta(T)$. The dimension of the range of T is called the rank of T and is denoted by $\rho(T)$.

Sum of Rank and Nullity: Let $T: U \rightarrow V$ be a L.T. from n-dimenal vector space U into V. Then sum of the dimensions of the range and kernel is equal to the dimension of the domain U. That is

$$\rho(T) + \eta(T) = n$$

or $$\dim(R(T)) + \dim(\ker T) = \dim(U)$$