Inner Product Space:

Let V be a vector space (real or complex) over a field F. Let $\alpha, \beta \in F$ and $u, v, w \in V$ be arbitrary. Then $\langle , \rangle$ is called an inner product space on V if the following are satisfied:-

(i) $\langle u, v \rangle \in F$

(ii) $\langle u, v \rangle = \langle v, u \rangle$ or $\langle u, v \rangle = \overline{\langle v, u \rangle}$ If F is complex (C)

The conjugate of $\langle u, v \rangle$

(iii) $\langle u, v+w \rangle = \langle u, v \rangle + \langle u, w \rangle$

$\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$

(iv) $\langle \alpha u, v \rangle = \alpha \langle u, v \rangle$

$\langle u, \alpha v \rangle = \bar{\alpha} \langle u, v \rangle$ $\because \bar{\alpha}$ is conjugate of α

(v) $\langle \alpha u + \beta v, w \rangle = \alpha \langle u, w \rangle + \beta \langle v, w \rangle$

(vi) $\langle u, u \rangle \geq 0$ i.e $\|u\| \geq 0$ and $\langle u, u \rangle = 0$ iff $u = 0$

The vector space V with an inner product is called an inner product space.

* A real inner product space is also called an Euclidean space and a complex inner product space is called an unitary space.

* A linear operator T on an inner product space $V(F)$ is called self-adjoint operator iff $T = T^{*}$ i.e $\langle T(u), v \rangle = \langle u, T(v) \rangle$

(i) A self-adjoint operator is symmetric if inner product space is Euclidian space ($F = R$)

(ii) A self-adjoint operator is Hermitian if IPS is Unitary ($F = C$)

For unitary operator $T: V(F) \to V(F)$

(i) T is one-one onto

(ii) $\langle T(u), T(v) \rangle = \langle u, v \rangle$ $\forall u, v \in V$