Infinite series:

If a set of real numbers $a_1, a_2, a_3 \cdots\cdots$ occur according to some definite law then

$a_1 + a_2 + a_3 + \cdots\cdots + a_n + \cdots$ is called a series.

(i) If the number of terms is limited, then the series is called to be a finite series

and (ii) if the number of terms is infinitely large, it is called an infinite series.

An infinite series is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots\cdots \infty \text{ or } \sum a_n$$

and the sum of first n terms is denoted by S_n

Thus.

$$S_n = a_1 + a_2 + \cdots\cdots + a_n$$

and a_n is the nth term of the series.

Clearly, S_n is a function of n and as n increases indefinitely three possibilities arise:-

(1) If S_n tends to a finite limit as $n \to \infty$, then the series $\sum a_n$ is said to be convergent

(2) If S_n tends to $\pm\infty$ as $n \to \infty$, the series is said to be divergent.

(3) If S_n does not tend to a unique limit as $n \to \infty$ then, the series $\sum a_n$ is said to be oscillatory or non-convergent.

Suppose Σa_n is an infinite series

We define a sequence $\langle S_n \rangle$ as follows

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots \qquad \vdots$$
$$S_n = a_1 + a_2 + a_3 + \cdots\cdots + a_n$$

The sequence $\langle S_n \rangle$ is called the sequ[ence] of partial sums of the series Σa_n.

Definition:- A series Σa_n is said to [be] convergent if the sequence $\langle S_n \rangle$ of partial sums of Σa_n is converg[ent].

If $\lim\limits_{n \to \infty} S_n = S$, then S is called the sum of the series Σa_n.

Write as:- $S = \sum\limits_{n=1}^{\infty} a_n$

→ The series Σa_n is said to be divergen[t] if the sequence $\langle S_n \rangle$ of partial sums of Σ is divergent.

Illustrations of series

1. $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$

2. $\sum_{n=1}^{\infty} \frac{x^n}{\lfloor n} = x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\cdots$

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\cdots$

4. $\sum_{n=1}^{\infty} (-1)^n n = -1 + 2 - 3 + 4 - 5 + \cdots\cdots$

5. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$