Indeterminate Forms: Thus, if any expression $\frac{f(x)}{g(x)}$ becomes of the form $\frac{0}{0}$ at the point $x=a$, then this form is called indeterminate i.e the value of $\frac{f(x)}{g(x)}$ at that point is indefinite.

Thus, types of Indeterminate forms are:-

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\times\infty$, $\infty-\infty$, $0^0$, $1^\infty$ and $\infty^0$.

Thus, the value of the indeterminate form $\frac{0}{0}$ is calculated by using L. Hospital rule.

Exp. Limit of $\frac{\sin\theta}{\theta}$ where $\theta \to 0$

or $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta}$ or $\underset{\theta\to 0}{Lt} \frac{\sin\theta}{\theta}$

Solution $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = \frac{\sin 0}{0} = \frac{0}{0}$

This is form of $\left[\frac{0}{0}\right]$ Indeterminate form

Then apply L. Hospital rule.

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \lim_{\theta\to 0} \frac{\left[\frac{d}{d\theta}\sin\theta\right]}{\left[\frac{d}{d\theta}\theta\right]} = \lim_{\theta\to 0} \frac{\cos\theta}{1}$$

$$= \lim_{\theta\to 0} \frac{\cos\theta}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

Q. Evaluate form $\frac{0}{0}$

$$\lim_{x \to 1} \frac{\log x}{x^n - 1}$$

Solution Given that

$$\lim_{x \to 1} \frac{\log x}{x^n - 1} \quad \text{applying limit}$$

$$= \frac{\log 1}{1 - 1} = \frac{0}{0}$$

This form is indeterminate form $\frac{0}{0}$ then using L. Hospital rule.

$$\lim_{x \to 1} \frac{\log x}{x^n - 1} = \lim_{x \to 1} \frac{\frac{d}{dx} \log x}{\frac{d}{dx}(x^n - 1)}$$

$$= \lim_{x \to 1} \frac{1/x}{n x^{n-1}} = \lim_{x \to 1} \frac{1}{n x^n}$$

$$= \frac{1}{n(1)^n} = \frac{1}{n}$$

$$= \lim_{x \to 1} \frac{\log x}{x^n - 1} = \frac{1}{n}$$