Heat

Rctilinear flow of Heat

Lecture - 2

Dr. Tarun Kumar Dey,
Professor in Physics
HOD ,Electronics
L.S College; BRA Bihar University, Muzzaffarpur

Online platform :http::/meet. findmementor.com

Rectilinear flow of Heat (Fourier Equation for flow of Heat) :

Let us consider a long metal bar of uniform cross –sectional area A, heated at one end .

The bar lies along the x – axis whose origin is at the hot end . Heat flows along this axis .

Now consider a thin slice with two parallel planes A and B perpendicular to the length of the bar at distances x and $x + \delta x$ from the hot end .

If θ be the temperature of the plane A

and $\frac{d\theta}{dx}$ be the heat which leaves the slice at B per second .

Then

$$Q = - K A \frac{d\theta}{dx} \qquad (1)$$

$$Q - dQ = - K A \frac{d}{dx} (\theta + \frac{d}{\theta dx} \delta x) , \qquad (2)$$

Where K is the thermal conductivity of the bar . The negative sign indicates that the temperature decreases with increases x .

Then the heat gained by the slice is

$$dQ = -\ K\,A\,\frac{d}{\theta dx}\ -\{\ -\ K\,A\ \frac{d}{dx}\,(\,\theta + \frac{d}{\theta dx}\,\delta x\,)\}\,,$$

$$= \ KA\frac{d2}{dx2}\delta x\ . \qquad (3)$$

If dQ be the amount gained by the slice is used partly in raising the temperature of the slice and partly in radiation from the surface of the slice .

Let ρ be the density and s be the specific heat of the material of the bar , then mass of the slice is

$$= A\delta x\ \rho$$

Let E be the emissive power of the surface and θbe the average excess temperature of the surface over the surrounding , then according to Newton's Law of Cooling , the heat radiated per second from the surface is

$$dQ_2 = E p \delta x \theta \qquad (5)$$

Hence we have

$$KA\, d^2\theta / dx^2\, \delta x = A \delta x \rho s \frac{d}{\theta dt} + E p \delta x \theta$$

This can be written as ,

$$\frac{d\theta}{dt} = h\, d^2\theta / dx^2 - \mu\theta , \qquad (6)$$

Where

$$h = \frac{K}{\rho s} \quad ,$$

$$\mu = \frac{E}{Aps} \qquad (7)$$

The constant h is known as the diffusivity of material , which determines the rate at which the temperature changes in a bar .

Equation (6) is the standard **Fourier equation for linear flow of heat** . If we neglect the heat lost by radiation , (6) reduces to

$$\frac{d\theta}{dt} = h\, d^2\theta \;/dx^2 \qquad (8)$$

Steady state

The temperature remains constant in slice element , this state is named as steady state . Hence

$$\frac{d\theta}{dt} = 0$$

and (4) reduces to

$$d^2\theta\ /dx^2 = \frac{\mu}{h}\theta = m^2\,\theta\ , \qquad (9)$$

where $m^2 = \frac{\mu}{h}\theta = \frac{Ep}{KA}$

Here the radiation is included .

It is a second order differential equation and can be solved to give

$$\theta = e^{\pm mx}$$

Hence complete solution of (9) is

$$\theta = A\, e^{mx} + B\, e^{-mx} \qquad (10)$$

Where A and B are constants whose values are determined from the boundary conditions .

Let us assume

$\theta = \theta_0$ at $x = 0$

$\theta = 0$ at $x = \propto$

Then

$\theta_0 = A + B$

$0 = A \ e^{\propto}$

But $e^{\propto}$ is not zero and hence A = 0 .

Thus the finding solution will be

$$\theta = \theta_0 \ e^{-mx} \qquad (11)$$

When bar is covered , so that the radiation loss is neglected ,(9) becomes

$$d^2\theta / dx^2 = 0 \qquad (12)$$

$$\theta = Ax + B \qquad (13)$$

Where A and B are constants , which can be determined again from the boundary conditions.

If l be the length of the bar , then

$\theta = \theta_0$ at $x = 0$

$\theta = \theta_1$ at $x = l$

From (13) we get

$B = \theta_0$

$A = \theta_0 - \theta_1 / l$

And hence

$\theta = \theta_0 - (\theta_0 - \theta_1 / l)\, x$

Where θ is the temperature at any point x .