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Hamiltonian Dynamics - 01

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Hamiltonian Dynamics

In Lagrangian formulation , the equations of motion for any system is in generalised coordinates q_1 , q_2 , …….q_n

but in Hamiltonian dynamics , used the generalised momenta in place of generalised velocity $\dot{q}_1$, $\dot{q}_2$ …… $\dot{q}_n$

used in the Lagrangian formulation.

In the Hamiltonian formulation , two set of first order differential equations are used instead of second order differential equations.

Both the formulations are equivalent , but the Hamiltonian is more fundamental to the foundations of statistical and quantum mechanics .

Generalised momentum and **Cyclic coordinates**

The generalised momentum corresponding to generalised coordinate q_k is defined as

$$p_k = \frac{\partial L}{\partial \dot{q}}$$

This is also called conjugate momentum or canonical momentum .

Lagrange's equations are given by

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_k}\right] - \frac{\partial L}{\partial q_k} = 0$$

Substituting for

$$p_k = \frac{\partial L}{\partial \dot{q}_k} ,$$

we get

$$\frac{dp_k}{dt} - \frac{\partial L}{\partial q_k} = 0$$

Or , $\dot{p}_k = \frac{\partial L}{\partial q_k}$

Now ,we consider in the expression for Lagrangian L of a system , a certain coordinate q_k does not appear explicitly , Then

$$\frac{\partial L}{\partial q_k} = 0$$

This means that

$$\dot{p}_k = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = 0$$

After integration . we get

$$p_k = \frac{\partial L}{\partial \dot{q}_k} = \text{Constant}$$

Whenever the Lagrangian function does not contain a coordinate q_k explicitly , the generalised momentum is a constant of motion .
The coordinate q_k is called cyclic or ignorable .

In other words , the generalised momentum associated with an ignorable coordinate is a constant of motion for the system .