Solution of Linear Simultaneous Equations:-

Simultaneous linear equations occur in Various physical problems. We know that a given system of linear equations can be solved by Cramer's rule or by matrix method. However, there exist other numerical methods of solution which are well-suited for computing machines.

* Direct methods of solution:-

In Gauss elimination method, the unknowns are eliminated successively and the system is reduced to an upper triangular system from which the unknowns are found by back substitution.

Working Rule for Gauss Elimination Method

Consider the Equations:-

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots\cdots a_{1n}x_n &= b_1 \text{ —(i)} \\ a_{21}x_1 + a_{22}x_2 + \cdots\cdots a_{2n}x_n &= b_2 \text{ —(ii)} \\ a_{31}x_1 + a_{32}x_2 + \cdots\cdots a_{3n}x_n &= b_3 \text{ —(iii)} \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots\cdots a_{nn}x_n &= b_n \text{ —(n)} \end{aligned}\right\} \text{ —— (1)}$$

Step 1- To Eliminate x_1 from second to n^{th} equations. Here, we take three Eqns of Eqn (1)

Assuming $a_{11} \neq 0$, we eliminate x_1 from the IInd Equation by subtracting (a_{21}/a_{11}) times the Ist Eqn from the IInd Eqn. Similarly we eliminate x_1 from the IIIrd Eq. by subtracting (a_{31}/a_{11}) times the Ist Eqn from the IIIrd Equn. We thus, get the new system:

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ + a'_{22}x_2 + a'_{23}x_3 &= b'_2 \\ + a'_{32}x_2 + a'_{33}x_3 &= b'_3 \end{aligned}\right\} \quad \text{---} (2)$$

Here, $a'_{22} = a_{22} - a_{12}\frac{a_{21}}{a_{11}}$ &

$a'_{23} = a_{23} - a_{13} \cdot \frac{a_{21}}{a_{11}}$ and Ist Eqn is called pivotal Equation then a_{11} is Ist pivot.

Step-2:- To Eliminate x_2 from IIIrd Eqn of (2).

Assuming $a'_{22} \neq 0$, we eliminate x_2 from the IIIrd Eqn of (2) by subtracting (a'_{32}/a'_{22}) times the IInd Eqn from IIIrd Eqns of (2), we get.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a'_{22}x_2 + a'_{23}x_3 &= b'_2 \\ a''_{33}x_3 &= b''_3 \end{aligned}\right\} \text{———} (3)$$

Here, the IInd Eqn is the pivotal Eqn and a'_{22} is the new pivot.

Step-3:- To evaluate the unknowns.

Thus the value of x_1, x_2, x_3 are found the reduced system (3) by back substitution.

Exp- Apply Gauss Elimination method to solve the equations

$$x + 4y - z = -5 \quad \text{--- (i)}$$

$$x + y - 6z = -12 \quad \text{--- (ii)}$$

$$3x - y - z = 4 \quad \text{--- (iii)}$$

Solve. (i) To eliminate x from (ii) and (iii) by operate as (ii) − (i) and (iii) − 3(i) then

$$-3y - 5z = -7 \quad \text{--- (iv)}$$

$$-13y + 2z = +19 \quad \text{--- (v)}$$

(ii) To Eliminate y from (v) by operate (v) − $\frac{13}{3}$ (iv)

$$\frac{71}{3} z = \frac{148}{3} \quad \text{--- (vi)}$$

from (vi) $z = \frac{148}{71}$

from (iv) $-3y - 5 \times \frac{148}{71} = -7$

$$y = \frac{7}{3} - \frac{5(148)}{71} = \frac{-81}{71}$$

from (i) =

$$x = -5 + 4\left(\frac{-81}{71}\right) + \frac{148}{71}$$

$$x = \frac{117}{71}$$

Hence, the solution of the linear system equations $x = 117/71$

$y = -81/71$

$z = 148/71$

Ans.