Mathematical Formulation of a L.P.P. :-

It is important to recognize a problem which can be handled by linear programming and then to formulate its mathematical model.

Example :-

(1) A manufacturer of a line of patent medicines is preparing plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45,000 bottles into which either of the medicines can be put. Further more it take 3 hours to prepare enough material to file 1,000 bottles of A. it takes one hour to prepare enough material to fill 1,000 bottles of B and there are 66 hours available for this operation. The profit is Rs 8/- per bottle for A and Rs 7/- per bottle of B.

(a) Formulate this problem as a linear programming problem?

(b) How should the manufacturer schedule production in order to maximize his profit?

(a) Let the manufacturer produce x_1 and x_2 bottles of medicines A and B respectively.

∴ Total profit (in Rs) $Z = 8x_1 + 7x_2$

The time required to prepare x_1 bottles of medicine A $= 3x_1/1000$ hours. ——— (1)

and the time required to prepare x_2 bottles of medicine B

$= x_2/1000$ hours

$\therefore$ Total time required to prepare x_1 bottles of medicine A and x_2 bottles of medicine B is $\frac{3x_1}{1000} + \frac{x_2}{1000}$ hours

Since total time available for this operation is 66 hours

$$\therefore \frac{3x_1}{1000} + \frac{x_2}{1000} \leq 66$$

or, $3x_1 + x_2 \leq 66000$

Since there are only 45000 bottles into which the medicines can be put

$\therefore x_1 + x_2 \leq 45000$

Hence the linear programming problem of the given problem is as follows.

Max. $Z = 8x_1 + 7x_2$

Subject to the Constraints

$3x_1 + x_2 \leq 66,000, x_1 + x_2 \leq 45000$

$x_1 \leq 20,000$ $\quad x_2 \leq 40000$

and $x_1 \geq 0$, $x_2 \geq 0$

Sol:- First Consider the Constraints as equations

Step one :-

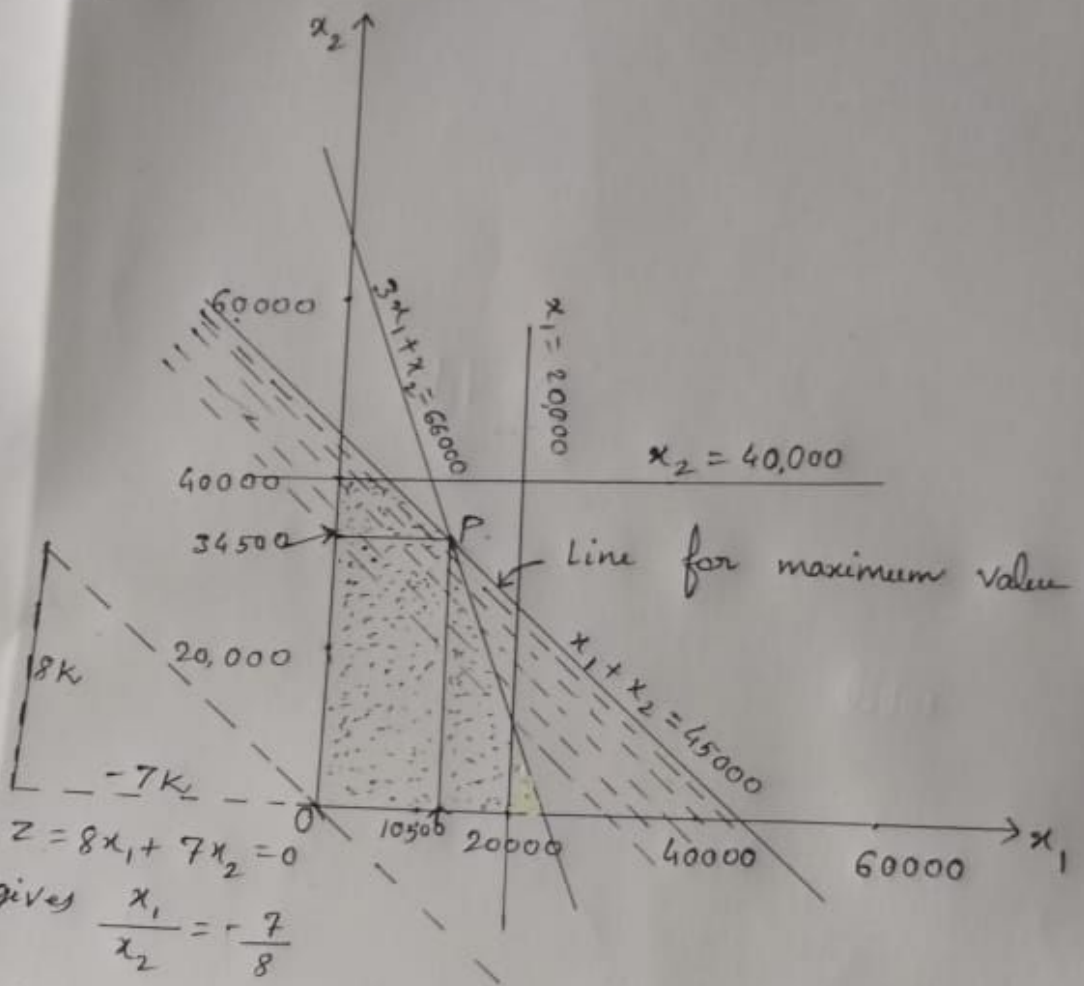
$3x_1 + x_2 = 66000$

$x_1 + x_2 = 45000$

$x_1 = 20000$

$x_2 = 40000$

step two :- Now draw the lines to each of the above equations in two dimensional plane



step three :- The shaded region shown in fig is the permissible region for the values of the variables x_1 and x_2.

step four :- To find x_1, x_2 for which Z is maximum, we draw the line (dotted line through 0) $Z = 8x_1 + 7x_2 = 0$ and Continue drawing parallel lines till we reach the point of the permissible region which is farthest away from the origin. As shown in figure

this point is at point P (10500, 34500) which is the point of intersection of lines.

$$3x_1 + x_2 = 66000$$

and $$x_1 + x_2 = 45000$$

$\therefore$ Z is maximum for $x_1 = 10,500$, $x_2 = 34,500$

and maximum $Z = 8 \times (10,500) + 7 \times (34,500)$

$= 3,25,500$ Ans.