Exp. Find the general solution of the diff. Equ.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{4x}\sin 2x.$$

Sol. Given the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{4x}\sin 2x$$

The general solution is

$$y = C.F + P.I$$

To find C.F from the homogeneous Equ.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$

The auxiliary equation is

$$D^2 - 4D + 3 = 0$$

$$(D-3)(D-1) = 0$$

$D = 3$ and $D = 1$ then

$$C.F = c_1 e^{3x} + c_2 e^{x}$$

To find P.I of $(D^2 - 4D + 3)y = e^{4x}\sin 2x$

$$P.I = \frac{1}{D^2 - 4D + 3} \cdot e^{4x}\sin 2x$$

$$= e^{4x} \cdot \frac{1}{(D+4)^2 - 4(D+4) + 3}\sin 2x$$

$$= e^{4x} \frac{1}{D^2 + 16 + 8D - 4D - 16 + 3}\sin 2x$$

$$P.I = e^{4x} \cdot \frac{1}{D^2+4D+3} \sin 2x$$

$$= e^{4x} \cdot \frac{1}{(-4)+4D+3} \sin 2x$$

$$= e^{4x} \cdot \frac{1}{4D-1} \sin 2x$$

$$= e^{4x} \cdot \frac{4D+1}{16D^2-1} \sin 2x$$

$$= e^{4x} \cdot \frac{4D+1}{16(-4)-1} \sin 2x$$

$$= e^{4x} \cdot \frac{4D+1}{-65} \sin 2x$$

$$= \frac{e^{4x}}{-65} [4 D \sin 2x + \sin 2x]$$

$$= -\frac{e^{4x}}{65} [4 \cdot \cos 2x \cdot 2 + \sin 2x]$$

$$P.I = -\frac{e^{4x}}{65} [8 \cos 2x + \sin 2x]$$

Thus the general solution is

$$y = C.F + P.I$$

$$y = c_1 e^{3x} + c_2 e^{x} + \frac{-e^{4x}}{65} [8 \cos 2x + \sin 2x]$$

$$y = c_1 e^{3x} + c_2 e^{x} - \frac{e^{4x}}{65} [8 \cos 2x + \sin 2x]$$