* Gauss-Seidal Iteration Method :-

This is a modification of the Jacobi's iteration method. As before we start with initial approximations x_0, y_0, z_0 (each $=0$) for x, y, z respectively. Substituting $y = y_0$, $z = z_0$ in the first (i) Equation of the Equations

$$\left.\begin{aligned} x &= k_1 - l_1 y - m_1 z \quad \text{--- (i)} \\ y &= k_2 - l_2 x - m_2 z \quad \text{--- (ii)} \\ z &= k_3 - l_3 x - m_3 y \quad \text{--- (iii)} \end{aligned}\right\} \text{--- (2)}$$

we get

$$x_1 = k_1$$

Then putting $x = x_1$, $z = z_0$ in the (ii) of the Eqn (2) we have-

$$y_1 = k_2 - l_2 x_1 - m_2 z_0$$

Next substituting $x = x_1$, $y = y_1$ in the (iii) of the Eqn (2) we get

$$z_1 = k_3 - l_3 x_1 - m_3 y_1$$

and so on i.e. as soon as new approximation for an unknown is found. it is immediately used in the next step.

This process of iteration is continued till convergency to be desired degree of accuracy is obtained.

Exp. Apply Gauss-Seidal iteration method to solve the equations.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Solution We write the given equtions in the form

$$\left.\begin{aligned} x &= \frac{1}{20}(17 - y + 2z) \quad \text{--- (i)} \\ y &= \frac{1}{20}(-18 - 3x + z) \quad \text{--- (ii)} \\ z &= \frac{1}{20}(25 - 2x + 3y) \quad \text{--- (iii)} \end{aligned}\right\} \text{--- (1)}$$

1) For the first iteration,

We start ~~from~~ the approximation $x_0 = y_0 = z_0 = 0$

Substituting $y = y_0$ and $z = z_0$ in the right side of the first (i) of epuations (1), we get

$$x_1 = \frac{1}{20}(17 - y_0 + 2z_0) = \frac{1}{20}(17 - 0 + 2\times 0) =$$

$$x_1 = \frac{17}{20} = 0.85$$

Putting $x = x_1$ and $z = z_0$ in the (ii) of Ep (1) we get

$$y_1 = \frac{1}{20}(-18 - 3x_1 + z_0)$$
$$= \frac{1}{20}(-18 - 3\times 0.85 + 0) = -1.0275$$

Putting $x = x_1$, $y = y_1$ in the third (ii) of the Ep (1) we have

$$z_1 = \frac{1}{20}(25 - 2\times 0.85 + 3\times(-1.0275)$$

$$z_1 = 1.0109$$

2) For the second iteration we have

$$x_2 = \frac{1}{20}(17 - y_1 + 2z_1) = \frac{1}{20}(17 - (-1.0275 + 2\times(1.0109))$$

$$x_2 = 1.0025$$

$$y_2 = \frac{1}{20}(-18 - 3x_2 + z_1) = \frac{1}{20}(-18 - 3\times(1.0025) + 1.0109)$$

$$y_2 = -0.9998$$

$$z_2 = \frac{1}{20}(25 - 2x_2 + 3y_2) = \frac{1}{20}\left\{25 - 2\times(1.0025) + 3\times(-0.9998)\right\}$$

$$z_2 = 0.9998$$

3) For the third iteration, we get

$$x_3 = \frac{1}{20}(17 - y_2 + 2z_2) = \frac{1}{20}(17 - (-.9998) + 2(0.9998))$$

$$x_3 = 1.0000$$

$$y_3 = \frac{1}{20}(-18 - 3x_3 + z_2) = \frac{1}{20}(-18 - 3\times(1.0) + 0.9998)$$

$$y_3 = -1.0000$$

$$z_3 = \frac{1}{20}(25 - 2x_3 + 3y_3) = \frac{1}{20}(25 - 2\times(1.0) + 3(1.00))$$

$$z_3 = 1.0000$$

The values in the 2nd and 3rd iterations being practically the same, we can stop iteration for next. Hence the solution of the epuations is —

$$x = 1.0 \quad y = -1.0 \text{ and } z = 1.0$$