

PG SEMISTR II,  
UNIT III,  
SYMMETRY  
ELEMENTS

BY

Dr. PRIYANKA

# Character Tables

Schoenflies symmetry symbol

$D_{3d}$	$E$	$2C_3$	$3C_2$	$i$	$2S_6$	$3\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	$R_z$ $(R_x, R_y)$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	-1	1	1	-1		$(x^2 - y^2, xy),$ $(xz, yz)$
$E_g$	2	-1	0	2	-1	0		
$A_{1u}$	1	1	1	-1	-1	-1	$z$ $(x, y)$	
$A_{2u}$	1	1	-1	-1	-1	1		
$E_u$	2	-1	0	-2	1	0		

Mulliken symbols

Characters of the irreducible representations

$x, y, z$   
 $R_x, R_y, R_z$

Squares & binary products of the coordinates

## Mulliken Symbol Notation

1) **A** or **B**: 1-dimensional representations

**E**: 2-dimensional representations

**T**: 3-dimensional representations

2) **A** = symmetric with respect to rotation by the  $C_n$  axis

**B** = anti-symmetric w/respect to rotation by  $C_n$  axis

Symmetric = + (positive) character

Anti-symmetric = - (negative) character

- 3) Subscripts **1** and **2** associated with **A** and **B** symbols indicate whether a  $C_2$  axis  $\perp$  to the principle axis produces a **symmetric (1)** or **anti-symmetric (2)** result.

$D_{3d}$	$E$	$2C_3$	$3C_2$	$i$	$2S_6$	$3\sigma_d$
$A_{1g}$	1	1	1	1	1	1
$A_{2g}$	1	1	-1	1	1	-1

If  $C_2$  axes are absent, then it refers to the effect of vertical mirror planes (e.g.,  $C_{3v}$ )

$C_{3v}$	$E$	$2C_3$	$3\sigma_v$
$A_1$	1	1	1
$A_2$	1	1	-1
$E$	2	-1	0

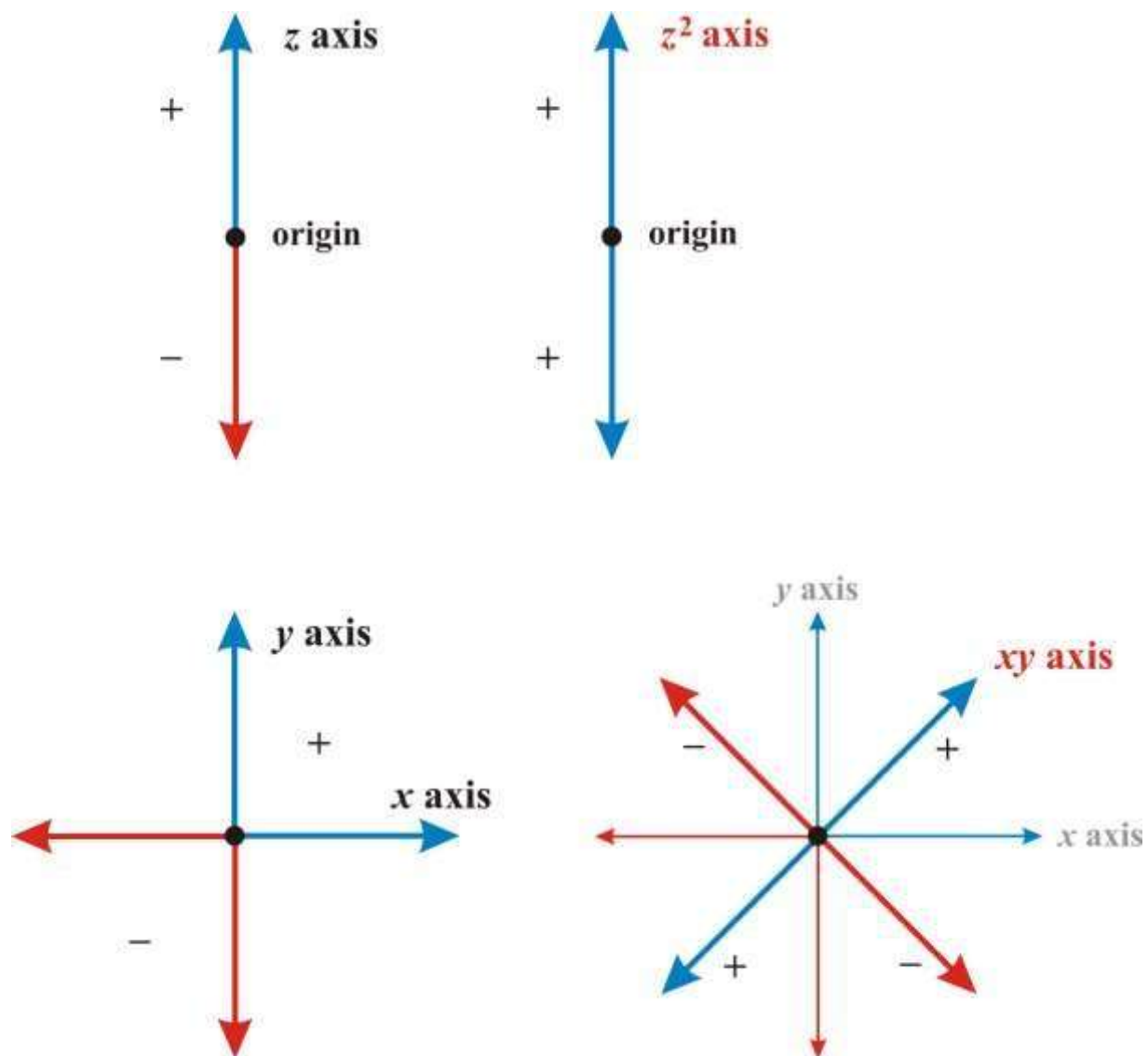
- 4) Primes and double primes indicate representations that are **symmetric ( ' )** or **anti-symmetric ( " )** with respect to a  $\sigma_h$  mirror plane. They are NOT used when one has an inversion center present (e.g.,  $D_{2nh}$  or  $C_{2nh}$ ).
- 5) In groups with an **inversion** center, the subscript “g” (“gerade” meaning even) represents a Mulliken symbol that is **symmetric** with respect to **inversion**.

The symbol “**u**” (“ungerade” meaning uneven) indicates that it is **anti-symmetric**.

- 6) The use of numerical subscripts on **E** and **T** symbols follow some fairly complicated rules that will not be discussed here. Consider them to be somewhat arbitrary.

## Square and Binary Products

These are higher order “combinations” or products of the primary  $x$ ,  $y$ , and  $z$  axes.



## The Great Orthogonality Theorem

$$\sum [\Gamma_i(\mathbf{R})_{mn}] [\Gamma_j(\mathbf{R})_{m'n'}]^* = \frac{h}{\sqrt{l_i l_j}} \delta_{ij} \delta_{mm'} \delta_{nn'}$$

- $\Gamma_i(\mathbf{R})_{mn}$  The element in the  $m^{\text{th}}$  row and  $n^{\text{th}}$  column of the matrix corresponding to the operation  $\mathbf{R}$  in the  $i^{\text{th}}$  irreducible representation  $\Gamma_i$ .
- $\Gamma_i(\mathbf{R})_{mn}^*$  complex conjugate used when imaginary or complex #'s are present (otherwise ignored)
- $h$  the order of the group
- $l_i$  the dimension of the  $i^{\text{th}}$  representation  
( $\mathbf{A} = 1$ ,  $\mathbf{B} = 1$ ,  $\mathbf{E} = 2$ ,  $\mathbf{T} = 3$ )
- $\delta$  delta functions, = 1 when  $i = j$ ,  $m = m'$ , or  $n = n'$ ; = 0 otherwise

The different irreducible representations may be thought of as a series of orthonormal vectors in  $h$ -space, where  $h$  is the order of the group.

Because of the presence of the delta functions, the equation = 0 unless  $i = j$ ,  $m = m'$ , or  $n = n'$ . Therefore, there is only one case that will play a direct role in our chemical applications:

$$\sum_R [\Gamma_i(\mathbf{R})_{mn}] [\Gamma_j(\mathbf{R})_{m'n'}] = 0 \quad \text{if } i \neq j$$

$$\sum_R [\Gamma_i(\mathbf{R})_{mn}] [\Gamma_j(\mathbf{R})_{m'n'}] = 0 \quad \begin{array}{l} \text{if } m \neq m' \\ n \neq n' \end{array}$$

$$\sum_R [\Gamma_i(\mathbf{R})_{mn}] [\Gamma_i(\mathbf{R})_{mn}] = \frac{h}{l_i}$$