Exp. To find n^{th} derivative of $\log(ax+b)$ & $\cos(ax+b)$

Sol. (1) n^{th} diff. coefficient of $\log(ax+b)$

Let $y = \log(ax+b)$

on diff. w.r.to x, successively, we get

$$\frac{dy}{dx} = y_1 = \frac{1}{(ax+b)} \cdot a = a(ax+b)^{-1}$$

$$\frac{d^2y}{dx^2} = y_2 = (-1) \cdot a^2 (ax+b)^{-2}$$

$$\frac{d^2y}{dx^2} = y_3 = (-1)(-2) a^3 (ax+b)^{-3}$$

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$$\frac{d^ny}{dx^n} = y_n = (-1)(-2)(-3) \cdots (-n+1) a^n (ax+b)^{-n} \quad \text{--- (1)}$$

Again diff. w.r.to x, we get

$$\frac{d^{n+1}y}{dx^{n+1}} = y_{n+1} = (-1)(-2)(-3) \cdots (-n+1)(-n) a^{n+1} (ax+b)^{-(n+1)}$$

Hence, y_{n+1} of the same form as y_n, given in Eqn (1)

Thus, Eqn (1) is true for $n = 1, 2, 3, \cdots$ and so on.

Therefore, by mathematical induction, Eqn (1) is true for every (+ve) integer n.

From Eqn (1) we can write as

$$y_n = (-1)^{n-1} (n-1)!\, a^n (ax+b)^{-n}$$

$$\boxed{\text{i.e. } y_n = \frac{(-1)^{n-1} \lfloor (n-1)\, a^n}{(ax+b)^n}}$$

(ii) n^{th} diff. coeff. of $(\cos ax+b)$

Let $y = \cos(ax+b)$

on differentiating w.r.to x successibly, we get

$y_1 = -a\sin(ax+b) = a\cos(ax+b+\pi/2)$

$y_1 = a\cos(ax+b+\pi/2)$ $\left[\because \cos(\frac{\pi}{2}+\theta) = -\sin\theta\right]$

$y_2 = -a^2\sin(ax+b+\pi/2) = a^2\cos(ax+b+\frac{\pi}{2}+\pi/2)$

$y_2 = a^2\cos(ax+b+2\pi/2)$

$y_3 = -a^3\sin(ax+b+2\frac{\pi}{2}) = a^3\cos(ax+b+\frac{2\pi}{2}+\pi/2)$

$y_3 = +a^3\cos(ax+b+3\frac{\pi}{2})$

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$y_n = a^n\cos(ax+b+n\frac{\pi}{2})$ ——— ——— ②

Again diff. w.r. to x, we get

$y_{n+1} = a^{n+1}\cos(ax+b+(n+1)\pi/2$

Hence, y_{n+1} is the same form as y_n given in Eq ②

It is trure for particular value of n.

Threfore, by mathematical induction, Eq^n ② is trure for every (+ve) integer n.

From Eq^n ② can write as

$$\boxed{y_n = a^n\cos\left[ax+b+\frac{n\pi}{2}\right]}$$

Exp. find y_n if $y = \sin^2 x \cos^2 x$

Solution Given

$$y = \sin^2 x \cos^2 x = (\sin x \cos x)^2$$

$$y = \left(\frac{2\sin x \cos x}{2}\right)^2$$

$$y = \frac{1}{4}(\sin 2x)^2 = \frac{1}{4}\sin^2 2x$$

$$y = \frac{1}{4}\left[\frac{1-\cos 4x}{2}\right]$$

$$y = \frac{1}{8} - \frac{1}{8}\cos 4x \quad \text{—— } (1)$$

We have $y = \cos(ax+b) \Rightarrow y_n = a^n \cos\left(ax+b+\frac{n\pi}{2}\right)$

Then from (1)

$$y_n = 0 - \frac{1}{8}4^n \cdot \cos\left(4x + \frac{n\pi}{2}\right)$$

$$y_n = -\frac{1}{4\times 2} \cdot 4^n \cos\left(4x + \frac{n\pi}{2}\right)$$

$$y_n = -\frac{1}{2}4^{n-1} \cos\left(4x + \frac{n\pi}{2}\right)$$

$$y_n = -2^{-1} \cdot 2^{2n-2} \cos\left(4x + \frac{n\pi}{2}\right)$$

$$y_n = -2^{2n-3} \cos\left(4x + \frac{n\pi}{2}\right)$$