Exp. If $p = x\cos\alpha + y\sin\alpha$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$ Prove that

$$p^{\frac{m}{m-1}} = (a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}}$$

Sol. Given curve

$$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1 \quad \text{--- (1)}$$

Diff. w.r.t. x, we get

$$\frac{m x^{m-1}}{a^m} + \frac{m y^{m-1}}{b^m} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b^m}{a^m} \cdot \frac{x^{m-1}}{y^{m-1}}$$

The equation of tangent at (x, y) is

$$Y - y = \frac{dy}{dx}(X - x)$$

$$Y - y = -\frac{b^m}{a^m} \cdot \frac{x^{m-1}}{y^{m-1}}(X - x)$$

$$\Rightarrow (X - x)\frac{x^{m-1}}{a^m} + (Y - y)\frac{y^{m-1}}{b^m} = 0$$

$$\frac{X x^{m-1}}{a^m} - \frac{x^m}{a^m} + \frac{Y y^{m-1}}{b^m} - \frac{y^m}{b^m} = 0$$

$$X\frac{x^{m-1}}{a^m} + Y\frac{y^{m-1}}{b^m} = \frac{x^m}{a^m} + \frac{y^m}{b^m}$$

$$X\frac{x^{m-1}}{a^m} + Y\frac{y^{m-1}}{b^m} = 1 \quad \text{--- (2)}$$

$$x\cos\alpha + y\sin\alpha = p \quad \text{--- (3)}$$

But $x\cos\alpha + y\sin\alpha = p$ touches the given curve at the point (x, y).

Therefore the two equations (II) & (III) represent the same equation.

Therefore the coefficients of x & y and the constant term will be proportional in both the equations (II) & (III) $x\frac{\cos\alpha}{p} + y\frac{\sin\alpha}{p} = 1$

$$\frac{x^{m-1}}{a^m} = \frac{\cos\alpha}{p} \Rightarrow x^{m-1} = \frac{a^m \cos\alpha}{p}$$

$$\Rightarrow x = \left(\frac{a^m \cos\alpha}{p}\right)^{1/m-1}$$

$$\frac{y^{m-1}}{b^m} = \frac{\sin\alpha}{p} \Rightarrow y^{m-1} = \frac{b^m \sin\alpha}{p}$$

$$y = \left(\frac{b^m \sin\alpha}{p}\right)^{\frac{1}{m-1}}$$

Now since (x, y) satisfies the equation

$$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$$

Therefore

$$\frac{\left(\frac{a^m \cos\alpha}{p}\right)^{\frac{m}{m-1}}}{a^m} + \frac{\left(\frac{b^m \sin\alpha}{p}\right)^{\frac{m}{m-1}}}{b^m} = 1$$

$$\frac{a^{\frac{m^2}{m-1}} (\cos\alpha)^{\frac{m}{m-1}}}{a^m \cdot p^{m/m-1}} + \frac{b^{\frac{m^2}{m-1}} \cdot (\sin\alpha)^{\frac{m}{m-1}}}{b^m \cdot p^{m/m-1}} = 1$$

$$\frac{a^{\frac{m}{m-1}} \cdot (\cos\alpha)^{\frac{m}{m-1}}}{p^{m/m-1}} + \frac{b^{\frac{m}{m-1}} \cdot (\sin\alpha)^{\frac{m}{m-1}}}{p^{m/m-1}} = 1$$

$$(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{m/m-1}$$

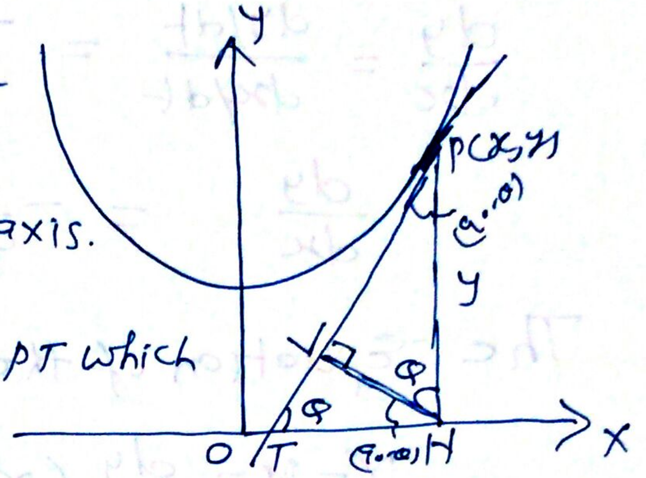
Proved.

Exp. Prove that for the ~~conte~~ catenary $y = c\cosh\frac{x}{c}$ the length of the perpendicular from the foot of the ordinate on the tangent is constant.

Sol. The given curve

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = c\left(-\sinh\frac{x}{c}\cdot\frac{1}{c}\right) = +\sinh\frac{x}{c}$$



Let P(x, y) be any point on the curve. From P, draw, PN ⊥^lar to x-axis.

Then PH = y

At P draw a tangent PT which meets the x axis at T and PT makes an angle θ with the x axis

i.e. ∠PTH = θ

From H, HL has been drawn ⊥^lar to the tangent PT.

We have to find length of HL.

from △TPN = 90 − θ & ∠LPH = 90 − θ

$$\sin(90-\theta) = \frac{HL}{PH}$$

$$\cos\theta = \frac{HL}{y} \Rightarrow HL = y\cos\theta$$

$$\tan\theta = \frac{dy}{dx} = +\sinh\frac{x}{c}$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = +\sinh\frac{x}{c}}$$

$$\because \cos\theta = \frac{1}{\sqrt{1+(\tan\theta)^2}} = \frac{1}{\sqrt{1+\sinh^2\frac{x}{c}}}$$

$$\Rightarrow \cos\theta = \frac{1}{\cosh\frac{x}{c}} = \frac{c}{y}$$

$$HL = y\cos\theta$$

$$HL = y\cdot\frac{c}{y} = c$$

HL = constant