Que. 1 Find the nth differential coefficient of $e^x \sin x \cdot \sin 2x$.

Solution: Let $y = e^x \sin x \cdot \sin 2x$

$$y = \frac{1}{2} e^x \cdot 2\sin x \cdot \sin 2x = \frac{1}{2} e^x \cdot 2\sin 2x \cdot \sin x$$

$$= \frac{e^x}{2}\left[\cos(2x - x) - \cos(2x + x)\right]$$

$$= \frac{1}{2} e^x \left[\cos x - \cos 3x\right]$$

$$y = \frac{1}{2}\left[e^x \cos x - e^x \cos 3x\right]$$

$$\therefore y_n = \frac{1}{2}\left[n^{th}\ d.c.\ of\ e^x \cos x - n^{th}\ d.c.\ of\ e^x \cos 3x\right]$$

We have $y = e^{ax} \cos bx \Rightarrow$

$$y_n = (a^2 + b^2)^{n/2}\left[e^{ax} \cos\left(bx + n \tan^{-1}\frac{b}{a}\right)\right]$$

$$y_n = \frac{1}{2}\left[(1^2+1^2)^{n/2} e^x \cdot \cos\left(x + n\tan^{-1}\frac{1}{1}\right) - (1^2+3^2)^{n/2} e^x \cdot \cos\left(3x + n\tan^{-1}\frac{3}{1}\right)\right]$$

$$y_n = \frac{1}{2} e^x \left[2^{n/2} \cdot \cos(x + n\tan^{-1} 1) - 10^{n/2} \cdot \cos(3x + n\tan^{-1} 3)\right]$$

$$y_n = \frac{e^x}{2}\left[2^{n/2} \cdot \cos(x + n\pi/4) - 10^{n/2} \cos(3x + n\tan^{-1} 3)\right]$$

Ans.

Que-2. If $y = A\sin mx + B\cos mx$, prove that $y_2 = -m^2y$

Sol: Given that

$$y = A\sin mx + B\cos mx \quad \text{--- (1)}$$

on differentiating Eqn (1) both sides w.r.to x

$$y_1 = mA\cos mx - mB\sin mx$$

Again diff. w.r. to x.

$$y_2 = -m^2A\sin mx - m^2B\cos mx$$

$$= -m^2(A\sin mx + B\cos mx)$$

$$y_2 = -m^2y \qquad \text{from (1)}$$

Que-3 If $y = \log(\sin x)$, prove that $y_3 = \frac{2\cos x}{\sin^3 x}$

Sol. Given that $y = \log(\sin x)$

Diff. w.r. to x

$$y_1 = \frac{1}{\sin x}\cdot\cos x = \frac{\cos x}{\sin x} = \cot x$$

$$y_1 = \cot x \Rightarrow y_2 = -\operatorname{cosec}^2 x = -\frac{1}{\sin^2 x}$$

$$y_2 = -(\sin x)^{-2}$$

Again diff. w.r.to x

$$y_3 = -(-2)\cdot(\sin x)^{-3}\cdot\cos x = \frac{2\cos x}{(\sin x)^3} = \frac{2\cos x}{\sin^3 x}$$