Exp. Find the singular solution of the differential equation $y = px + p - p^2$

Sol. Given that

$$y = px + p - p^2 \quad \text{———} \quad (1)$$

Differentiating w.r.to x, we get

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + \frac{dp}{dx} - 2p\frac{dp}{dx}$$

$$p = p + \frac{dp}{dx}(x+1-2p)$$

$$\frac{dp}{dx}(x+1-2p) = 0$$

$\therefore \frac{dp}{dx} = 0$ or $x+1-2p = 0$

if $\frac{dp}{dx} = 0 \Rightarrow dp = 0$ then $\int dp = c$

$$p = c$$

then general solution is

$$y = cx + c - c^2$$

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When $x+1-2p=0$

$$2p = x+1$$

$$p = \frac{x+1}{2}$$

Putting the value of p in Eqn (1) then we get

$$y = \frac{x+1}{2}\cdot x + \frac{x+1}{2} - \frac{(x+1)^2}{4}$$

$$4y = 2x^2+2x+2x+2-(x^2+1+2x)$$

$$= 2x^2+4x+2-x^2-1-2x$$

$$= x^2+2x+1$$

$$4y = (x+1)^2$$

$y = \frac{(x+1)^2}{4}$ which is singular solution.

$$\frac{(x+1)^2}{4} = cx+c-c^2$$

$$\frac{(x+1)^2}{4} = c(x+1-c)$$

$$(x+1)^2 = 4c(x+1-c)$$

$$x+1 = \pm 2\sqrt{c(x+1-c)}$$

$$x = -1 \pm 2\sqrt{c(x+1-c)}$$

$$y = \frac{(x+1)^2}{4}$$

Exp. Find the singular solution of the differential equation.

$$y = y'x + a\sqrt{1+y'^2}$$

Solution Given that

$$y = y'x + a\sqrt{1+y'^2} \quad \text{——— (i)}$$

$y' = p = \frac{dy}{dx}$ putting in Eqn (i)

$$y = px + a\sqrt{1+p^2} \quad \text{——— (ii)}$$

Differentiating w.r.to x Eqn (ii)

$$\frac{dy}{dx} = p + x\cdot\frac{dp}{dx} + a\cdot\frac{1}{2}\cdot\frac{1}{(1+p^2)^{1/2}}\cdot 2p\frac{dp}{dx}$$

$$p = p + x\frac{dp}{dx} + \frac{ap}{\sqrt{1+p^2}}\frac{dp}{dx}$$

$$\frac{dp}{dx}\left(x + \frac{ap}{\sqrt{1+p^2}}\right) = 0$$

Either $\frac{dp}{dx} = 0$ or $x + \frac{ap}{\sqrt{1+p^2}} = 0$

When $\frac{dp}{dx} = 0 \Rightarrow p = c$

Then general solution is from Eqn (ii)

$$y = cx + a\sqrt{1+c^2} \quad \text{——— (iii)}$$

When $x + \frac{ap}{\sqrt{1+p^2}} = 0 \Rightarrow x = \frac{-ap}{\sqrt{1+p^2}}$ i.e. $x^2(1+p^2) = a^2p^2$

$$a = \frac{-x\sqrt{1+p^2}}{p}$$

$$p^2 = \frac{x^2}{a^2-x^2} \Rightarrow p = \frac{x}{\sqrt{a^2-x^2}}$$

Putting p in Eqn (ii)

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$y = px + a\sqrt{1+p^2}$

Putting $a = \frac{-x\sqrt{1+p^2}}{p}$ then

$y = px + \left(\frac{-x}{p}\sqrt{1+p^2}\right).\left(\sqrt{1+p^2}\right)$

$= px - \frac{x(1+p^2)}{p}$

$y = \frac{p^2x - x - p^2x}{p} = \frac{-x}{p}$

$y = -\frac{x}{p} = -\sqrt{a^2-x^2}$ from $p = \frac{x}{\sqrt{a^2-x^2}}$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2$

This is singular solution

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