Exp. Solve the Equation

$$y''+2y'+2y = e^{x} x \sin x$$

Sol. Given that second order non-homogeneous linear differential equation

$$y''+2y'+2y = e^{x} x \cdot \sin x \quad \text{--- (1)}$$

Here. Homogeneous differential Equation is

$$y''+2y'+2y=0$$

and

$$f(x) = e^{x} \cdot x \sin x$$

Then. the auxiliary Equation is

$$m^2+2m+2=0$$

$$m = \frac{(-2) \pm \sqrt{4-4\times2\times1}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$m = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$m_1 = -1+i, -1-i$$

From $f(x) = e^{x} \cdot x \sin x$, Here $\alpha = 1$ and $\beta = 1$ since $\alpha + i\beta = 1+i$ is not solutions of the auxiliary equation we can try a particular solution

$$y_p = e^{x}[(Ax+B)\sin x + (Cx+D)\cos x]$$

$$y_p = e^{x}[Ax \sin x + B \sin x + Cx \cos x + D \cos x]$$

On Differentiating its twice time and putting in Eqn (1). and solve.

Exp. Find a particular solution of the Equation.

$$y^{iv} + y'' = x^3 + x^2 + e^x(x+1) + 2\cos x$$

Sol. The corresponding homogeneous equation from (1)

$$y^{iv} + y'' = x^3 + x^2 + e^x(x+1) + 2\cos x \quad \text{--- (1)}$$

$$y^{iv} + y'' = 0 \quad \text{--- (2)}$$

and $f(x) = x^3 + x^2 + e^x(x+1) + 2\cos x$

Then, the auxiliary equation is

$$m^4 + m^2 = 0$$

$$m^2(m^2+1) = 0$$

$m = 0, 0$ and $m^2 = -1 \Rightarrow m = \pm i$

Thus the roots are $m = 0, 0, +i, -i$. Here $\alpha = 0$ and $\beta = 1$ thus $1, x, \cos x$ are solutions of the reduced equation. We put. $\alpha + i\beta = 0 + i$

$$y_p = x^2(Ax^3 + Bx^2 + Cx + D) + e^x[Ex + F] + x[H\sin x + G\cos x]$$

$$y_p = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ee^x x + Fe^x + Hx\sin x + Gx\cos x \quad \text{--- (3)}$$

on differentiating Eqn (3) w.r.t. to x

$$y' = 5Ax^4 + 4Bx^3 + 3Cx^2 + 2Dx + E(e^x \cdot 1 + x \cdot e^x) + Fe^x + G(-x\sin x + \cos x) + H(x\cdot\cos x + \sin x)$$

$$y'' = 20Ax^3 + 12Bx^2 + 6Cx + 2D + E(e^x + x\cdot e^x + e^x) + Fe^x + G(-x\cos x - \sin x - \sin x) + H(-x\sin x + \cos x + \cos x)$$

$$y'' = 20Ax^3 + 12Bx^2 + 6Cx + 2D + E(2e^x + x.e^x) + Fe^x - Hx\sin x - G(x\cos x + 2\sin x)$$

Again differentiating

$$y''' = 60Ax^2 + 24Bx + 6C + E(2.e^x + e^x.1 + x.e^x) + Fe^x - H(x\cos x + \sin x) - G(x.(-\sin x) + \cos x \ldots)$$

Again

$$y^{iv} = 120Ax + 24B + E(3.e^x + e^x + x.e^x) + Fe^x - H(-x\sin x + \cos x + \cos x) + G[x.\cos x + \sin x + 3\sin x]$$

$$y^{iv} = 120Ax + 24B + E(4e^x + x.e^x) + Fe^x + Hx.\sin x - 2H\cos x + Gx\cos x - 2G\sin x$$

on substituting y^{iv} and y'' in Eq^n (1)

$$120Ax + 24B + E(4e^x + x.e^x) + Fe^x + Hx\sin x - 2H\cos x + Gx\cos x - 2G\sin x + 20Ax^3 + 12Bx^2 + 6Cx + 2D + E(2e^x + x.e^x) + Fe^x - Hx\sin x - Gx\cos x + 2G\sin x = e^x(x+1) + 2\cos x + x^3 + x^2$$

$$20Ax^3 + 12Bx^2 + x(120A + 6C) + 24B + 2D + e^x(2Ex + 6E + 2F) + 4G\sin x - 2H\cos x = x^3 + x^2 + e^x x + e^x + 2\cos x.$$

$20A = 1 \Rightarrow A = 1/20$ $\quad 2E = 1 \Rightarrow E$

$12B = 1 \Rightarrow B = 1/12$ $\quad -2H = 2 \Rightarrow H =$

$120A + 6C = 0 \Rightarrow C = -1$ $\quad 4G = 0 \Rightarrow G =$

$24B + 2D = 0 \Rightarrow D = -1$ $\quad 6E + 2F = 1 \Rightarrow F$

Putting values of A, B, C, D, E, F, G and H in Eqn (iii) then we have a particular solution. is

$$y_p = \frac{1}{20}x^5 + \frac{1}{12}x^4 + (-1)x^3 + (-1)x^2 + e^x \cdot x(1/2) - e^x$$

$$(-1)x\sin x$$

$$y = \frac{1}{20}x^5 + \frac{1}{12}x^4 - x^3 - x^2 + e^x\left(\frac{1}{2}x - 1\right) - x\sin x$$

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