Exp. Using Newton Raphson method, find the root of the equation $x + \log_{10} x = 3.375$ correct to four significant figures.

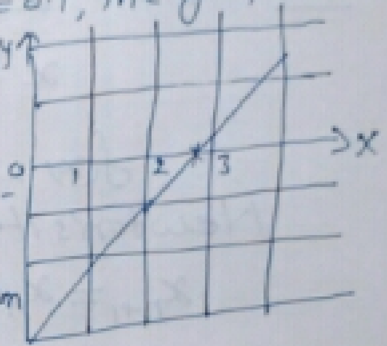
Solution Let

$$y = x + \log_{10} x - 3.375$$

To obtain a rough estimate of its root, we draw the graph of (i) with the help of the following table.

x	1	2	3	4
y	-2.375	-1.074	0.102	1.227

Taking 1 unit along either axis $= 0.1$, the graph is as shown in fig. The curve crosses the x axis at $x_0 = 2.9$, which we take as the initial approximation to the root.



Now let us apply Newton-Raphson method to

$$f(x) = x + \log_{10} x - 3.375$$

$$\therefore f'(x) = 1 + \frac{1}{x} \log_{10} e$$

$$\therefore f(2.9) = 2.9 + \log_{10} 2.9 - 3.375 = -0.0126$$

$$f(2.9) = 2.9 + 0.462398 - 3.375$$

$$f'(2.9) = 1 + \frac{1}{2.9} \log_{10} e = 1.1497$$

The first approximation x_1 to be root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.9 + \frac{0.0126}{1.1497} = 2.9109$$

$$f(x_1) = 2.9109 + \log_{10}(2.9109) - 3.375 = -0.0001$$

$$f'(x_1) = 1 + \frac{1}{2.9109} \log_{10} e = 1.1492$$

Thus the second approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.9109 + \frac{0.0001}{1.1492} = 2.91099$$

Hence the required root, correct to four significant is 2.911

Exp. Evaluate $\sqrt{28}$ to four decimal places by Newton's iterative method.

Sol. Let $x = \sqrt{28}$

$$x^2 = 28 \Rightarrow x^2 - 28 = 0 \quad \text{---(i)}$$

$$f(x) = x^2 - 28$$

Newton's iterative method gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{2x_n} = \frac{1}{2}\left(x_n + \frac{28}{x_n}\right) \quad \text{---(ii)}$$

Now since $f(5) = -3$, $f(6) = 8$, a root of (i) lies betwee 5 & 6.

Taking $x_0 = 5.5$ (ii) gives

$$x_1 = \frac{1}{2}\left(x_0 + \frac{28}{x_0}\right) = \frac{1}{2}\left[5.5 + \frac{28}{5.5}\right]$$

$$x_1 = 5.29545$$

$$x_2 = \frac{1}{2}\left[x_1 + \frac{28}{x_1}\right] = \frac{1}{2}\left[5.29545 + \frac{28}{5.29545}\right] = 5.2915$$

$$x_3 = \frac{1}{2}\left[x_2 + \frac{28}{x_2}\right] = \frac{1}{2}\left[5.2915 + \frac{28}{5.2915}\right] = 5.2915$$

Hence $x_2 = x_3 = 5.2915$ upto 4 decimals

So $\sqrt{28} = 5.2915$