Linear Combination

Linear Combination - A vector v in a vector space V over field F, is called a linear combination of the vectors $v_1, v_2, v_3 \dots v_n$ in V. If vector v can be written in the form

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots \alpha_n v_n \qquad v_i \in V \text{ and } \alpha_i \in F.$$

where $\alpha_1, \alpha_2 \dots \alpha_n$ are scalars.

Exp. The vector $v = (1,1,1)$ as a linear combination of vectors in the set S.

$S = \{(1,2,3), (0,1,2), (-1,0,1)\}$ finding the linear combination.

Sol. The set of vectors $S = \{\underset{v_1}{(1,2,3)}, \underset{v_2}{(0,1,2)}, \underset{v_3}{(-1,0,1)}\}$

Let $\alpha_1, \alpha_2, \alpha_3$ are scalars of field F.

Then $v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ — ①

$$(1,1,1) = \alpha_1(1,2,3) + \alpha_2(0,1,2) + \alpha_3(-1,0,1)$$

$$(1,1,1) = (\alpha_1 + 0 - \alpha_3,\ 2\alpha_1 + \alpha_2 + 0,\ 3\alpha_1 + 2\alpha_2 + \alpha_3)$$

$$\left.\begin{aligned} \alpha_1 - \alpha_3 &= 1 \quad \text{— (i)} \\ 2\alpha_1 + \alpha_2 &= 1 \quad \text{— (ii)} \\ 3\alpha_1 + 2\alpha_2 + \alpha_3 &= 1 \quad \text{— (iii)} \end{aligned}\right\} \text{② system of linear Eqn.}$$

Using Gauss-Jordon Elimination, this system is non-homogeneous and it has infinite solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 1 \end{array}\right]$$

let $\alpha_3 = t$

$\alpha_1 = 1 + t$

$\alpha_2 = -2t - 1$

Put $t = 1$ then $\alpha_1 = 2$, $\alpha_2 = -3$, & $\alpha_3 = 1$

Thus the linear combination from ①

$$v = 2v_1 - 3v_2 + v_3$$

⇒ If possible, write the vector $v = (1,-2,2)$ as a linear combination of vectors in the set S from above example.

* Linear Dependence and Independence of Vectors:

Vectors (matrices) $V_1, V_2, V_3 \cdots V_n$ are said to be dependent if

(i) All the vectors (row or column matrices) are of the same order.

(ii) All n scalars $\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_n$ (not all zero) exist such that

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \cdots \alpha_n V_n = 0$$

Otherwise they are linearly independent

(iii) if $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_n = 0$.

Exp. Examine for linear dependence [1, 0, 2, 1] [3, 1, 2, 1], [4, 6, 3 -4], [-6, 0, -3, -4] and find the relation between them, if possible.

$$\sum_{i=1}^{4} \alpha_i V_i = \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \alpha_4 V_4 = 0$$

$$\alpha_1 (1, 0, 2, 1) + \alpha_2 (3, 1, 2, 1) + \alpha_3 (4, 6, 3 -4) + \alpha_4 (-6, 0, -3, -4) = 0$$

$$\alpha_1 + 3\alpha_2 + 4\alpha_3 + (-6)\alpha_4 = 0$$

$$0 \cdot \alpha_1 + \alpha_2 + 6\alpha_3 + 0 \cdot \alpha_4 = 0$$

$$2 \cdot \alpha_1 + \alpha_2 + 6\alpha_3 + 0 \cdot \alpha_4 = 0$$

$$\alpha_1 + \alpha_2 - 4\alpha_3 - 4\alpha_4 = 0$$