Exp. Test for convergence the series

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots$

Sol. (i) given series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$$

So that

$$\frac{1}{2} < 1, \quad \frac{1}{3} < \frac{1}{2}, \quad \frac{1}{4} < \frac{1}{3}, \ldots\ldots$$

$\because$ $a_{n+1} < a_n \quad \forall n$,

and $\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1}{n} = 0$

Hence, by Leibnitz's Test the given series is convergent but not absolutely convergent.

$\because$ A series $\sum a_n$ is said to be absolutely convergent if the series $\sum |a_n|$ is convergent, $\sum |a_n| = 1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n}$ this series in not convergent.

(ii) Given series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}$$

So that

$$\frac{1}{\sqrt{2}} < 1, \quad \frac{1}{\sqrt{3}} < \frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{4}} < \frac{1}{\sqrt{3}}, \ldots$$

(i) $\because a_{n+1} < a_n \ \forall n$, each term is numerically less than its preceding term.

(ii) $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{1}{\sqrt{n}}$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right)^{1/2} = 0$$

Hence, by Leibnitz's rule (Test) the given series is convergent.

Exp. Examine the character of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n-1} = 1 - \frac{2}{3} + \frac{3}{5} - \frac{4}{7} + \cdots$

Sol. Given Series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n-1} = 1 - \frac{2}{3} + \frac{3}{5} - \frac{4}{7} + \cdots\cdots$$

So that $\frac{2}{3} < 1$, $\frac{3}{5} < \frac{2}{3}$, $\frac{4}{7} < \frac{3}{5}$, ... and so on

$\because$ $a_{n+1} < a_n \ \forall n$ i.e. each term is numerical less than its preceding term.

& $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2n-1} = \lim_{n\to\infty} \left(\frac{1}{2-1/n}\right)$

$= \frac{1}{2} \neq 0$

$\lim_{n\to\infty} a_n \neq 0$ which is not zero

Hence the given series is not convergent. it is oscillatory.