Exp. Find a real root of the equation $x^3 = 1 - x^2$ on the interval $[0,1]$ with an accuracy of 10^{-4}.

Solution We rewrite the equation as

$$x = \frac{1}{\sqrt{x+1}} \quad \text{—— (i)}$$

Here

$$\phi(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$$

Therefore, $\phi'(x) = -\frac{1}{2}(x+1)^{-3/2} = -\frac{1}{2\sqrt{(x+1)^3}} < 1$ in $[0,1]$

Also, $\max|\phi'(x)| = \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}} = k < 0.2$

$\left[\because \epsilon \le \frac{1}{1000},\ \because \delta \le \frac{1}{100}\right.$

Therefore, Eqn gives

$$|x_n - x_{n-1}| \le \frac{1-0.2}{0.2}\epsilon = 4 \times 10^{-4} = 0.0004$$

Taking $x_0 = 0.75$, we find

$$x_1 = \frac{1}{\sqrt{0.75+1}} = \frac{1}{\sqrt{1.75}} = 0.75593$$

$$x_2 = \frac{1}{\sqrt{0.75593+1}} = \frac{1}{\sqrt{1.75593}} = 0.75465$$

$$x_3 = \frac{1}{\sqrt{0.75465+1}} = \frac{1}{\sqrt{1.75465}} = 0.75493$$

Now $|x_3 - x_2| = 0.00028 < 0.0004$. Hence, the required root is 0.7549, correct to four decimal places.

Exp. Use the iterative method to find a real root of the equation $\sin x = 10(x-1)$. that root lies between 1 and π with correct to three decimal places.

Solution- Let $f(x) = \sin x - 10x + 10$

We rewrite the equation as

$$x = 1 + \frac{\sin x}{10}$$

We have

$$\phi(x) = 1 + \frac{\sin x}{10}$$

$$\phi'(x) = \frac{\cos x}{10} < 1 \quad \text{in } 1 \le x \le \pi$$

$$|\phi'(x)| = \frac{\cos x}{10} < 1 \quad \text{in } 1 \le x \le \pi$$

Taking $x_0 = 1$, we obtain the successive iterates as

$$x_1 = 1 + \frac{\sin x_0}{10} = 1 + \frac{\sin 1}{10} = 1 + \frac{0.841}{10} = 1.0841$$

$$x_2 = 1 + \frac{\sin x_1}{10} = 1 + \frac{\sin(1.0841)}{10} = 1 + \frac{0.884}{10} = 1.0884$$

$$x_3 = 1 + \frac{\sin x_2}{10} = 1 + \frac{\sin(1.0884)}{10} = 1 + \frac{0.886}{10} = 1.0886$$

$$x_4 = 1 + \frac{\sin x_3}{10} = 1 + \frac{\sin(1.0886)}{10} = 1 + \frac{0.886}{10} = 1.0886$$

Hence the required correct root is 1.089

Exercise-1 Use the method of iteration to find a positive root of the equation $xe^{x} = 1$, given that a root lies between 0 and 1.

Exercise-2. Find a real root, correct to three decimal places, of the equation $2x - 3 = \cos x$ lying in the interval $\left[\frac{3}{2}, \pi/2\right]$.