Solvable Equations : General solution of Non-linear diff.

1- Equation is solvable for $y'(x)$. Solving Eqn $F(x,y,p)=0$ —① for p leads

$$y'(x) = f_i(x,y) \quad i = 1,2,\cdots l \quad —(2)$$

where l is a number of solution.

Exp. Find the general solution of the diff. Eqn

$$y'^2 - 4xy = 0 \quad \text{at } xy>0$$

Sol. Given that

$$y'^2 - 4xy = 0$$

$$y' = \pm 2\sqrt{xy}$$

$$y' = 2\sqrt{xy} \quad \text{and} \quad y' = -2\sqrt{xy}$$

Integrating, we find that

$$y' = 2\sqrt{xy}$$

$$\frac{dy}{dx} = 2\sqrt{xy}$$

$$\int y^{-1/2} dy = \int 2 x^{1/2} dx + c$$

$$2y^{1/2} = 2\cdot\frac{x^{3/2}}{3/2} + c$$

$$y^{1/2} = \frac{2}{3}x^{3/2} + c_1$$

$$y = \left(\frac{2}{3}x^{3/2} + c_1\right)^2$$

and

$$y' = -2\sqrt{xy}$$

$$\frac{dy}{dx} = -2\sqrt{xy}$$

$$\int y^{-1/2}\,dy = -2\int x^{1/2}\,dx + c$$

$$2y^{1/2} = -2\frac{x^{3/2}}{3/2} + c$$

$$y = \left(-\frac{2}{3}x^{3/2} + c_2\right)^2$$

where c_1 and c_2 are arbitrary constants.
Putting $c_2 = -c_1 = c$, we obtain the general solution

$$y = \left(\frac{2}{3}x^{3/2} + c\right)^2$$

The given equation also has the singular solution $y=0$

Exercise → Find the general solution of the non-Linear diff. Eq^n.

$$8y'^3 - 27y = 0$$

Exercise → Find the general solution of the non-Linear Eq^n.

$$y'^2 - 4x = 0$$